NETWORK UTILITY MAXIMIZATION IN AD HOC NETWORKS WITH DIFFERENT COMMUNICATION PATTERNS

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Abstract – **In this paper, we study joint end-to-end congestion control and medium access control in ad-hoc networks with fixed wireless channels as a utility maximization problem with constraints that arise from contention for channel access. We analyze the throughput of ad hoc networks with different network interaction models at communication level. Two types of network areas are being considered: square and rectangle. Our results show that full-mesh and small-world models have highest throughput, while star and scale-free models have lowest throughput. The effect of the network area depends on the network model but generally the square area gives slightly higher throughput.**

Keywords – **Ad hoc wireless network, complex network, congestion control, medium access control and network utility maximization**

1. INTRODUCTION

After the publication of the seminal paper [1], the network utility maximization (NUM) framework has found applications in network resources allocation through congestion control protocols. In the NUM framework, each end-user has its utility function and link bandwidths allocated so that network utility is maximized. Network utility can be represented as sum of all users' utilities, but some other definitions are also possible. A utility function can be interpreted as the level of satisfaction attained by a user as a function of resource allocation. Different kinds of utility functions lead to different types of fairness. A family of utility functions is proposed in [2].

In this paper we consider the problem of congestion control in a multihop wireless ad hoc network. Mobile ad hoc networks have been an active research area over the past years with many attractive and complex issues like: mobility, channel estimation, power control, medium access control (MAC), routing, etc. Unlike in wire-line networks where links are disjoint resources with fixed capacities, in ad hoc wireless networks the link capacities are "elastic" [3]. Most routing schemes for ad hoc networks select paths that minimize hop count. This implicitly predefines a route for any source-destination pair of a static network, independent of the pattern of traffic demand and interference among links. This may result in congestion at some region, while other regions are not fully utilized. To use the wireless spectrum more efficiently, multiple paths based on the pattern of traffic demand and interference among links should be considered [4].

Wireless channel is a shared medium and interference-limited. Link is only a logical concept and links are correlated due to the interference with each other. Under the MAC strategies such as time-division and random access, these links contend for exclusive access to the physical channel. Unlike in the wire-line network where network layer flows compete for transmission resources only when they share the same link, in wireless network flows can compete even if they don't share a wireless link in their paths. Thus, in ad hoc wireless networks the contention relations between link-layer flows provide fundamental constraints for resource allocation.

TCP congestion control algorithms can be considered as distributed primal-dual algorithms which maximize aggregate network utility, where a user's utility function is (often implicitly) defined by its TCP algorithm, [1] and [5]. These works implicitly assumes a wire-line network where link capacities are fixed and shared by flows that traverse common links. In wireless networks the joint design of congestion and media access control is naturally formulated using the network utility maximization framework considering the new constraints that arise from channel contention. There are many alternative decompositions of these algorithms, leading to a choice of different layering architectures. In [6] a survey is given of the current status of horizontal decomposition into distributed computation, and vertical decomposition into functional modules such as congestion control, routing, scheduling, random access, power control, and channel coding.

We focus on congestion control at the transport layer and channel contention at the MAC layer, and ignore all other issues. This is an active research area and many works are focused on these issues. For the MAC layer, in [3] and [7], random access is considered; while in others scheduling and multi-channel networks are considered [4]. In this paper we use scheduling because it gives better view of the maximal network throughput.

Ad hoc networks are usually formed by people in order to share information, and people are also part of some social network. Since most human communication takes place directly between individuals, such networks are crucially important for communications. Different ways of modeling users and their actions in the ad hoc network is presented in [8]. User interactions can be modeled as a complex network. A complex network is a network with non-trivial topological features – features that do not occur in simple networks such as lattices or random graphs. The study of complex networks is a young and active area of scientific research inspired largely by the empirical study of real-world networks such as computer networks and social networks [9], [10]. Two well-known and much studied classes of complex networks are scale-free [11] and small-world [12], [13] networks.

The main goal in this paper is to investigate the maximum end-to-end throughput of ad hoc networks with different interaction models at communication level, in order to get a better view of what is the expected maximum throughput of these networks when used in real-world situations.

The rest of the paper is organized as follows. In Section 2, we give a mathematical representation of wireless network and define the network utility maximization problem with its constraints and utility function. After that, in Section 3, we define the performance metric and present the different network models and the network areas which we use. Simulation results and analysis are given in Section 4, and Section 5 concludes the paper.

2. MATHEMATICAL REPRESENTATION OF WIRELESS NETWORK

We consider an ad hoc wireless network represented by an undirected graph $G(V,E)$, where V is the set of nodes and E is the set of logical links. Each source node s has its utility function $U_s(x_s)$, which is a function of its transmitting data rate x_s and we assume it is continuously differentiable, increasing, and strictly concave. For its communication, each source uses a subset $L(s)$ of links. We define S as the set of all sources and $S(l)$ as the subset of sources that are traversing link l. We assume static topology (the nodes are in a fixed position). Also, each link has finite capacity c_l when it is active, i.e. we implicitly assume that the wireless channel is fixed or some underlying mechanism masks the channel variation.

Wireless transmissions are interference-limited. All links transmit at rate c_l for the duration they hold the channel. We assume that nodes can communicate with at most one other node at any given time, because node cannot transmit and receive simultaneously. Two links will interfere with each other, if either the sender or the receiver of one of the links is within the interference range of the sender or receiver of the other link. This is because we use TCP connections so the receiver has to send acknowledgment packets. For dealing with the contention among links, we use time-division multiple access, in other words we will use scheduling as in [4].

The contention relations between the links of the network can be captured with a flow contention graph (see [3], [14]). In the contention graph, each vertex represents an active link, and an edge between two vertices denotes the contention between the corresponding links: two links interfere with each other and cannot be active at the same time. Given a contention graph, we can identify all its maximal cliques. A maximal clique of a graph is a maximal complete sub-graph of the graph. Maximal cliques are local constructions and capture the local contention relations of the flows. Flows within the same maximal clique cannot transmit simultaneously, but flows in different maximal cliques may transmit simultaneously.

Finding all maximal cliques in a graph is NP-complete problem [15]. The time required to solve these problems using any currently known algorithm increases very quickly as the size of the problem grows. So even for moderately large versions of many of these problems (as ours is), we need extremely long time for solving the problem. Approximate method for finding maximal cliques in ad hoc wireless networks is given in [16]. We use a slightly different method in which we discretely scan the whole area looking for cliques formed around every scanning point; and then by comparison eliminate those which are not maximal cliques. With mq we denote a single maximal clique and with MQ the set of all maximal cliques. The fraction of time that each link can transmit is $f(l)$. So the sum of these fractions of all the links in one maximal clique cannot be greater than one.

As a utility function, we use a function which provides proportional fairness among the end users:

$$U(x_s) = \log x_s \tag{1}$$

This function is strictly concave, because the second derivative is negative. From the concavity of the utility function it follows that the optimal rates $\{\hat{x}_s\}$ satisfy the following condition:

$$\sum_s \frac{x_s - \hat{x}_s}{\hat{x}_s} \le 0, \tag{2}$$

This means that if rate of one transmitter rises, the rate of another transmitter will drop, and the drop will be proportionally larger than the rise. This property is known as the law of diminishing returns.

An unfair resource allocation is also possible, in which the goal is maximizing the overall throughput without any consideration about the fairness among the end users. The unfair utility function would be:

$$U(x_s) = x_s \quad (3)$$

Finally, we have the following utility maximization problem:

$$\max_{x_s} \sum_{s \in S} U_s(x_s) \quad (4)$$

subject to:

$$\sum_{s \in S(l)} x_s \le c_l f(l), \quad \forall l \in E$$

$$\sum_{l \in mq} f(l) \le 1, \quad \forall mq \in MQ$$

After some reformulations and relaxations, this problem can be decomposed both horizontally and vertically. Vertically it can be decomposed in separate TCP and MAC layer algorithms, and these algorithms can further be horizontally decomposed and solved in distributed manner as in [5] and [1]. For our analysis these decompositions are not needed, because we are interested in overall network performance, so we solve the problem in a centralized manner.

3. SIMULATION METHODOLOGY

3.1 Performance metric

In order to represent the performance of ad hoc network we use the maximum end-to-end throughput (MT) as performance indicator. MT is the total amount of bits received by all nodes per second and is measured in Mega bits per second (Mbps):

$$MT = \sum_{s \in S} x_s \quad (5)$$

3.2 Network models

At physical level we simulate the wireless ad hoc network by placing the nodes geographically random. While at communication level for the communications among the end users, we consider several types of network models: random, scale-free, small-world, star, geographically random and full-mesh.

3.2.1 Random model

The *random model* can be considered as the most basic model of complex networks. A random network is obtained by starting with a set of *n* vertices and randomly adding edges between them. Most commonly studied is the Erdős–Rényi model [17], denoted *G(n, p)*, in which every possible edge occurs independently with probability *p*. The degree distribution p_k (the fraction of nodes having *k* links) is a Poisson distribution.

3.2.2 Small-world model of Watts and Strogatz

Many real world networks exhibit what is called the small world property, i.e. most vertices can be reached from the others through a small number of edges. This characteristic is found, for example, in social networks [18], where everyone in the world can be reached through a short chain of social acquaintances. Another property of many networks is the presence of a large number of loops of size three, i.e. if vertex *i* is connected to vertices *j* and *k*, there is a high probability of vertices *j* and *k* being connected. For example, in a friendship network, if B and C are friends of A, there is a high probability that B and C are also friends. Networks with abundance of these short loops are said to have clustering effect. The most popular model of random networks with small world property and clustering effect was developed by Watts and Strogatz and is called the Watts-Strogatz *small-world model* [19]. The degree distribution for small-world networks is similar to random networks, with a peak $\langle k \rangle = 2\kappa$. Besides social networks, many other real networks exhibit the small world properties, including gene networks, neural networks, road maps, and electric power grids.

3.2.3 Scale-free networks of Barabási and Albert

After Watts and Strogatz's model, Barabási and Albert showed that the degree distribution of many real systems is characterized by an uneven distribution [20]. Instead of the vertices of these networks having a random pattern of connections with a characteristic degree, as with the ER and WS models, some vertices are highly connected while others have few connections, with absence of a characteristic degree.

More specifically, the degree distribution has been found to follow a power law for large *k*, $P(k) \sim k^{-\gamma}$, where γ is a constant whose value is typically in the range $2<\gamma<3$, although occasionally it may lie outside these bounds. Networks with these characteristics are called *scale-free* networks. In these networks typically there are only several nodes called hubs, which are highly connected, and many others with only few connections mainly towards some of the hubs. Many real networks appear to be scale-free, including the Internet, the World Wide Web, protein networks, citation networks, and some social networks.

3.2.4 Geographical Models

Complex networks are generally considered as lying in an abstract space, where the position of vertices has no particular meaning. For several kinds of networks, such as protein-protein interaction networks or networks of movie actors, this consideration is reasonable. However, there are many networks where the position of vertices is particularly important as it influences the network evolution. This is the case of ad hoc networks, or highway networks, where the position of wireless nodes and cities can be localized in a map and the edges between correspond to real physical entities, such as wireless links and roads. This kind of networks is called *geographical* or *spatial* networks. In geographical networks, the existence of a direct connection between nodes can depend on a lot of constraints such as the distance between nodes in ad hoc networks. The models developed for representting these networks should consider these constraints. One way to generate geographical networks is to distribute N nodes at random and link them with a given probability which decays with the distance. This model generates a Poisson degree distribution as observed for random graphs and can be used to model ad hoc networks.

3.2.4 Star and full mesh model

Star networks are one of the most common computer networks. In the real-world they are usually found as a leaf networks, mainly as local area networks. Star network consists of one central node, to which all other end nodes are connected.

Full mesh networks are type of networks in which each of the nodes is connected to every other node in the network. In the real-world they are usually found as a core of large networks.

3.3 Network area

From aspect of the shape, we consider two types of network areas: square and rectangle. It is natural to expect that the MT gained in these two areas will differ.

We compare the throughput gained in the two areas by the ratio of the MTs attained:

$$MT(r/s) = \frac{MT(rectangle)}{MT(square)} \quad (6)$$

4. SIMULATIONS AND RESULTS

We use CVX [21] for solving our network utility maximization problem defined with (4). CVX is a modeling system for disciplined convex programming (DCP). DCP is a methodology for constructing convex optimization problems and is meant to support the formulation and construction of optimization problems that the user intends from the outset to be convex. DCP imposes a set of conventions or rules. Problems which follow the rules can be rapidly and automatically verified as convex and converted to solvable form. Some problems can be reformulated to be made convex and then solved by appropriate methods for convex problems.

For our simulations, we use ad hoc network with 100 wireless nodes spread out in 1000x1000m square area and 500x2000m rectangle area. For some aspects of our simulations we use the IEEE 802.11b standard. Therefore, each of the logical links has a capacity of 2Mbps, the interference distance between the nodes is 550m and the communication distance is 250m. Fixed routing is used with the path with minimum hops between the sender and the receiver chosen as a route. Most of the real routing protocols used in ad hoc networks (e.g. DSR and AODV) use this kind of fixed routing. The network has one channel that means the whole bandwidth is used for all communications.

As we said, we will analyze the throughput of different ad hoc networks (ad hoc networks with different interaction model at communication level). First we consider the throughput in square area by using proportionally fair allocation (Fig. 1). It can be seen that the star network has the lowest MT. This is because the central node participates in all communications, so a bottleneck is formed. The maximum MT which we can expect in star network is equal to the capacity of the links (2Mbps); it can be attained only if the central node is active (transmits or receives) all the time. But in some of the communications the leaf node needs to go through several physical hops till it reaches the central node, so contention can occur between transmissions in which the central node doesn't participate, and those in which it does. Because we use proportional fairness at least some transmission time will be allocated for all the transmissions, so the central node will have to be inactive some time and the maximum MT can not be attained. If we use unfair allocation (Fig. 2), the maximum MT is achieved which is two times higher than the MT attained using proportional fairness. The downside here is that the central node has the exclusive right to transmit all the time, so some of the communications will not be realized.

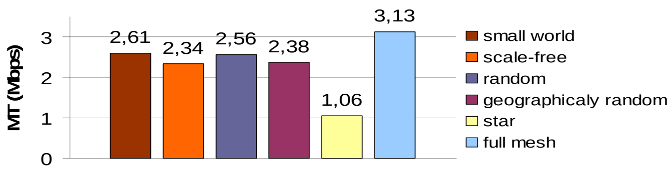

Fig. 1 MT of different ad hoc networks in square area by using proportionally fair utility function

Scale-free networks have the second worst throughput. In this network there are several hubs with high connectivity with which most of the nodes communicate. Similar explanation can be given as for the star network; with the difference that now we have several bottlenecks (at every hub), so the congestion is in some way relaxed. If one hub is inactive, because of some non-hub transmission, there is a high probability that some other hub is active. With unfair allocation the MT is more than two times higher, because more than one hub is present.

Small-world networks have higher throughput than scale-free network, because there are no bottlenecks present in it. Throughputs of random and geographically random networks are somewhere in between. Again, the unfair allocation raises the throughput of small-world, random and geographically random networks more than two times.

Full-mesh networks have significantly greater throughput than all other networks. In full-mesh network every node communicates with every other node, so there are many mutually non-contending communications. By allocating more time to these communications, high throughput is achieved.

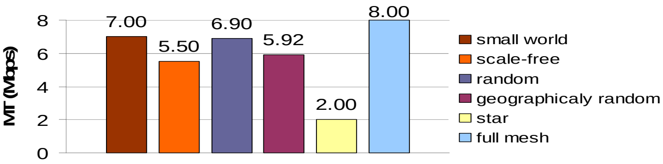

Fig. 2 MT of different ad hoc networks in square area by using unfair utility function

It can be noticed that the throughputs gained with the unfair allocation are two to three times greater than the throughputs gained with proportional fairness, but the relations between the throughputs of the different ad hoc networks are same.

Now we consider the throughput in rectangle area by using proportionally fair allocation (Fig. 3). The relations among the throughputs of the different ad hoc networks are same as in square area, but generally the throughputs are slightly lower.

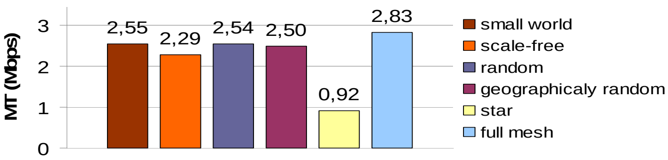

Fig. 3 MT of different ad hoc networks in rectangle area by using proportionally fair utility function

The ratios of the throughputs of the different ad hoc networks between rectangle and square area defined with (6) are shown on Fig. 4. We see that there is decrease in the throughput in rectangle area for all the networks except for the geographically random network. Because in rectangle area most of the nodes are placed further away than in square area, the average number of hops per communication increases. More hops mean more contention for channel access, and so the throughput decreases. The rectangle area is wider, so this will slightly reduce the interference, but not enough to compensate for the greater number of hops per communication.

On the other side, in the geographically random networks, nodes communicate only with nearby nodes, so the number of hops per communication doesn't increase. So because the rectangle area is wider with less interference, the throughput increases.

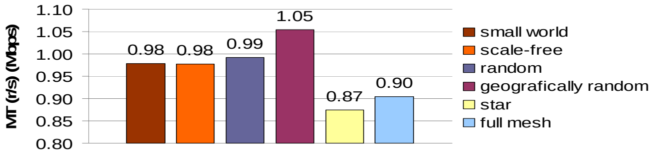

Fig. 4 Ratios of MTs of ad hoc networks with rectangle and square areas by using proportionally fair utility function

5. CONCLUSION

This paper has studied the throughput of ad hoc networks with different communication models using the network utility maximization framework.

We showed that full-mesh network has highest throughput. Small-world, random and geographically random networks have lower throughput than full-mesh network. Scale-free and star networks have lowest throughput, because of the bottlenecks at the hubs. The effect of the network area depends on the network model but generally the square area gives slightly higher throughput than rectangle area, because the average number of hops per communication is lower in square area.

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