Time series analysis and long range correlations of Nordic spot electricity market data

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Abstract

The electricity system price of the Nord Pool spot market is analysed. Different time scale analysis tools are assessed with focus on the Hurst exponent and long range correlations. Daily and weekly periodicities of the spot market are identified. While space time separation plots suggest stationary behaviour of the time series, we find large fluctuations of the spot price market which suggest time-dependent scaling parameters.

Key words: Hurst exponent, nonlinear time series analysis, long range correlations
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1 Introduction

The complex behaviour of financial time series has been the object of a considerable amount of studies [1,2]. It has been demonstrated that linear stochastic models are not able to capture properly this complexity and therefore it has

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been attributed to the fact that financial markets are nonlinear stochastic, chaotic or a combination of both. Specifically, in the last decades there has been a considerable amount of discussion about the characterisation of financial time series using the theory of Brownian motion [3,4], fractional Brownian motion [5], nonlinearity [6], chaos and fractals [7–9], scaling behaviour [10,11], and self organised criticality [12,13]. Most of the tests developed in the area of economic theory provide evidence of nonlinear dynamics, which may be deterministic or not deterministic. There is no convincing evidence of deterministic low-dimensionality in price series [14,15], and the claims of low-dimensional chaos have never been well-justified [16,17,11]. Nevertheless, in the last few years nonlinear time series analysis has expanded rapidly in the fields of Economics and Finance. This is also due to the fact that economic and financial time series seem to provide a promising area for the development, testing and application of nonlinear techniques [18] and the fact that high frequency financial time series are readily available.

Among these time series, energy spot prices have been analysed with several nonlinear techniques. Weron and Przybylowicz [19] studied the electricity prices using Hurst R/S analysis and showed anti-persistent behaviour with a Hurst exponent lower than 1/2. Using another technique, the average wavelet coefficient method [20], a Hurst exponent with value $H = 0.41$ has been obtained which is in agreement with other energy spot price time series. The question of modelling electricity spot prices has also been addressed by several researchers. Because of the high volatility in Nord Pool electricity prices, Byström [21] applied extreme value theory to investigate the tails of the price change distribution and then used the peaks-over-threshold method to analyse the data that exceed the threshold. Perelló et al. [22] proposed a GARCH model for the spot price. Weron et al. [23] fit a jump diffusion and regime switching model to Nord Pool spot prices. Vehviläinen and Pyykkönen [24] developed a stochastic factor based approach to mid-term modelling of spot prices taking into account climate data, hydro-balance, base load supply and the underlying mechanisms in spot price generation. The model was able to provide simulated values for the fundamental data, demand and supply information, and pricing strategies.

Here we are mainly concerned with quantifying long range correlations in energy spot price market data in terms of Hurst exponents. Such a concept has been widely used for the analysis of economic time series at the level of different quantities. For instance, Simonsen [25] analyses the volatility of the Elspot electricity market. Volatility clustering is observed, and relations between electricity markets and traditional financial markets are described. Main differences to traditional financial markets are a general high level of volatility and a possible dependence of the volatility on the price itself. Ref. [26] contains a brief discussion on the application of standard financial tools to electricity markets. In particular, it appears that the price for electricity is more volatile compared to other commodities because electricity cannot be stored in an efficient way. The Spanish electricity market is analysed in [27], using multifractal detrended fluctuation analysis. The Hurst exponent is estimated to $H = 0.16 \pm 0.01$. Ref. [28] analyses different energy prices, using the detrended moving average technique. In particular, crude oil, natural gas, heating oil, unleaded gasoline, and propane gas are considered. Focus is on the decay process of shocks in the return process.

We investigate in detail correlation properties of the Nord Pool electricity market. Some basic features about the time series are reviewed in section 2. In addition, we address the question whether such data can be described as a stationary process, at least on the time scales covered by the data set. To keep the presentation self-contained, we recall in section 3 some basic facts about the Hurst exponent and algorithms for the estimation of such a quantity. We then compare in section 4 results obtained by these different algorithms and evaluate in more detail fluctuation properties related with such exponents. We check, in particular, if surrogate time series with the same power spectrum but originated by a linear Gaussian process may have the same Hurst exponent. Some comments on large fluctuations are contained in the conclusion, section 5.

2 Data set and time series analysis

The Nordic electricity market, known as Nord Pool (http://www.nordpool.no) was created in 1993 and is owned by the two national grid companies, Statnett SF in Norway (50%) and Affärverket Svensa Kraftnät in Sweden (50%). The market was established as a consequence of the decision in 1991 by the Norwegian parliament to deregulate the market for power trading. Therefore, between 1992 and 1995 only Norway contributed to the market, in 1996 a joint Norwegian-Swedish power exchange was started-up and the power exchange was renamed Nord Pool ASA. Finland started a power exchange market of its own, EL-EX, in 1996 and joined Nord Pool in 1997. Beginning of 15th June 1998, Finland became an independent price area on the Nord Pool Exchange. The western part of Denmark (Jutland and Funen) has been part of the Nordic electric power market since 1 July 1999, whereas the eastern part of Denmark entered after 1st October 2000. On 5th October 2005 also the German area KONTEK was added in the Nord Pool exchange market.

The spot market operated by Nord Pool is an exchange market where participants trade power contracts for physical delivery the next day. Thus, it is referred to as a day-ahead market. The spot market is based on an auction with bids for purchase and sale of power contracts of one hour duration covering the 24 hours of the following day. At the deadline for the collection of

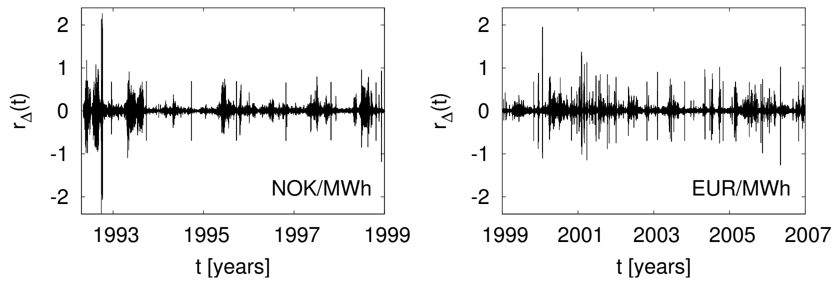

Fig. 1. Left: Hourly logarithmic return, Eq. (1), for the spot prices in the Nordic electricity market (Nord Pool) from May 1992 until December 1998. Right: Hourly logarithmic return for the spot prices from January 1997 until January 2007.

all buy and sell orders the information is gathered into aggregate supply and demand curves for each power-delivery hour. From these supply and demand curves the equilibrium spot prices – referred to as the system prices – are calculated.

We have analysed hourly data from the Nord Pool system spot prices. The series is divided into two parts. The first part, from 4th May 1992 to 31st December 1998, comprises 58,392 data points. The prices are indicated in Norwegian Krone (NOK)/MWh, whereas the second part of time series, from 1st January 1999 to 26th January 2007, comprises 70,752 data points with prices being expressed in EUR/MWh. We have considered the time series $s(t)$ as well as the corresponding returns over the time horizon Δ, defined as

$$r_\Delta(t) = \ln(s(t)/s(t-\Delta)) \,. \tag{1}$$

Figure 1 shows the hourly returns for the two parts of the time series considered.

Most of the probability theory of time series analysis is concerned with stationary processes. Broadly speaking a time series is said to be stationary if there is no systematic change in mean or in variance, i.e. no trend, and, if strictly periodic variations have been removed. However, it is also worth stressing that the non-stationary components, such as the trend, may sometimes be of more interest than the stationary residual. We here report on a relatively simple stationarity test, called space time separation plot [29]. For this purpose one evaluates the probability

$$P(r, \Delta t) = \mathrm{Prob}(\|x(t+\Delta t) - x(t)\| < r) \tag{2}$$

that phase space points, separated in time by an interval Δt, have distance less than r. If the process is stationary and if Δt exceeds the correlation time then such a quantity becomes independent of the time lag Δt and coincides with the correlation integral which is frequently used for estimating fractal properties of

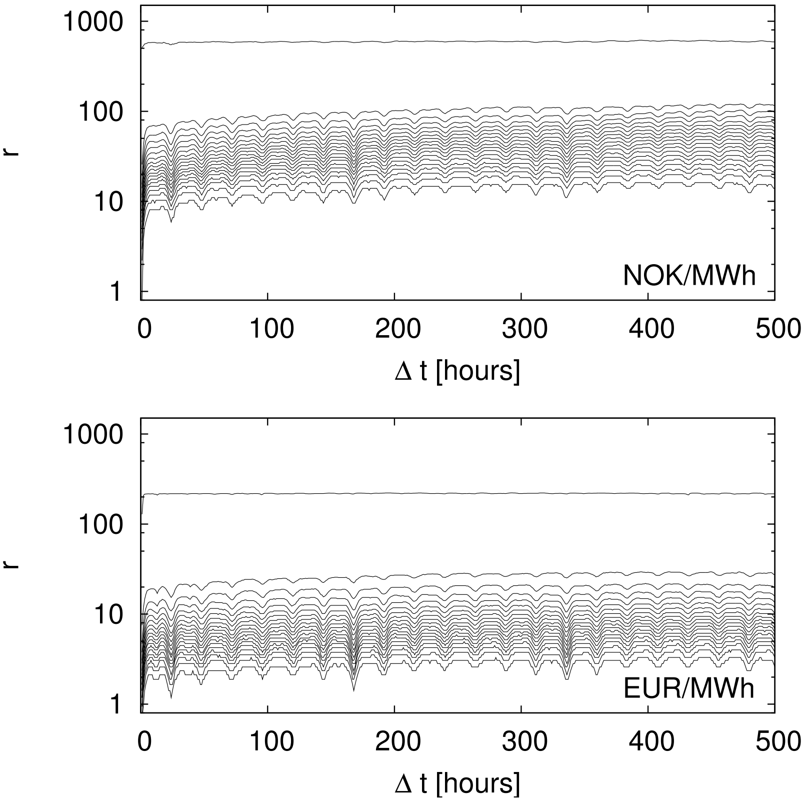

Fig. 2. Top: Space-time separation plot of the Nord Pool spot prices (NOK/MWh). Bottom: Space-time separation plot of the Nord Pool spot prices (EUR/MWh).

chaotic attractors. Since phase space coordinates are normally not accessible one employs standard delay embedding techniques to estimate the required probability function. We have used the program *stp* of the Tisean software package [30] which returns level lines of $P(r, \Delta t)$ for $P = 0.05, 0.1, 0.15, \ldots$. Horizontal level lines in such a contour plot indicate the required independence on Δt and are thus a signature of a stationary time series. Figure 2 shows the results of the test for analysed time series. In those graphics the separation time Δt is represented in the horizontal axis whereas the the separation in space, r, is represented in the vertical axis. As can be observed from Fig. 2 the contour plot obtained from the Nord Pool time series consists essentially of horizontal lines apart from a weak 24 hour periodicity. Thus $P(r, \Delta t)$ is essentially independent of Δt, unlike for instance, the high frequency exchange rates reported in [31]. The results shown in Fig. 2 support the assumption that the data set we are dealing with can be treated like a stationary process. Further evidence for stationarity comes from the observation that the contour plot does not change qualitatively when being based on shuffled surrogate data.

3 Long range correlations

An algebraic decay of the autocorrelation function $C(\tau) = \langle r_\Delta(t) r_\Delta(t+\tau)\rangle \sim |\tau|^{-\beta}$ on large time scales or the corresponding power law behaviour of the power spectrum $S(\omega) \sim |\omega|^{\beta-1}$ in the low frequency domain may be characterised in terms of the Hurst exponent $H = 1 - \beta/2$. A power law scaling of the correlation function on small time scales can be related with the fractal dimension of the corresponding stochastic process, and both quantities, the fractal dimension of the process and the Hurst exponent are in general independent quantities [32].

A tool for studying long-term memory and fractality of a time series is the rescaled range or R/S analysis first introduced by Hurst [33] in hydrology. Mandelbrot [34] argued that R/S analysis is a more powerful tool in detecting long range dependence when compared to more conventional methods like autocorrelation analysis, variance ratios, and spectral analysis. The range R of a time series with a finite sampling rate is defined by

$$R(\tau) = \max_t \left(X(t,\tau)\right) - \min_t \left(X(t,\tau)\right), \tag{3}$$

where $X(t,\tau)$ denotes the sum of the deviation of the time series $s(t)$ from its mean value $\langle s\rangle_\tau$ over some time interval τ

$$X(t,\tau) = \sum_{\ell=t}^{t+\tau} \left(s(\ell) - \langle s\rangle_\tau(t)\right) . \tag{4}$$

Moreover, $S(\tau)$ denotes the standard deviation of the time series over the time window τ. Computed for different sizes of the time window, the rescaled range $R(\tau)/S(\tau)$ shows a power law scaling

$$R(\tau)/S(\tau) \sim \tau^H \tag{5}$$

with exponent H. The Hurst exponent is equal to $1/2$ for Brownian motion, while $H < 1/2$ or $H > 1/2$ indicate anti-correlated and positively correlated increments, respectively.

Improved methods to estimate the Hurst exponent have been proposed to take care of non stationary components of the time series. The detrended moving average (DMA) uses the scaling behaviour of the standard deviation

$$\sigma_{DMA}(\tau) = \sqrt{\frac{1}{N-\tau} \sum_{\ell=\tau}^{N} \left(s(\ell) - \langle s\rangle_\tau(\ell)\right)^2} \tag{6}$$

about a moving average $\langle s \rangle_\tau(t)$ of a time series $s(t)$ of length N for different sizes τ of the moving average window (cf. [35,36]). The power law scaling of this standard deviation with the window size, $\sigma_{DMA}(\tau) \sim \tau^H$, yields the Hurst exponent. The generalised multifractal detrended fluctuation analysis (MF-DFA) pursues a similar idea. Here, the standard deviation is computed with regards to a low-order polynomial fit of the time series. One divides the time series $s(t)$ into n non-overlapping windows of equal size τ. For each window a polynomial fit to the time series is computed. The standard deviation

$$\sigma_{DFA}(\tau) = \sqrt{\frac{1}{N} \sum_{\ell=1}^{N} \left(s(\ell) - s_\tau^{poly}(\ell) \right)^2} \tag{7}$$

quantifies the variation of the time series $s(t)$ about the polynomial fit $s_\tau^{poly}(t)$ where the order m of the polynomials determines the order of the MF-DFA. Different orders differ in their ability to eliminate trends in the time series; see e.g. [37,38]. Again, the scaling of the standard deviation with the window size yields the Hurst exponent, $\sigma_{DFA}(\tau) \sim \tau^H$. The approach has been generalised by introducing a spectrum of Hurst exponents to take multifractal properties of the time series into account as well.

4 Results

We now report on the calculation of the Hurst exponent for the Nordic Pool Spot data using some of the methods just described. First, we have used the standard scaled windowed variance method [39] to estimate the Hurst exponent by linear regression of $\ln(R/S)$ versus $\ln(\tau)$. The results for the two original time series and surrogate series are shown in table 1. As it can be seen both parts of the time series show anti-persistence, $H < 1/2$. This has already been found by several researchers [19,20,22], amongst others. We generated two types of surrogate time series. The first type are Gaussian linear stochastic processes with the same mean, variance and power spectrum of the original data, the second are obtained by a random shuffling of the original time series. In all cases, the Hurst exponents of the original time series are slightly higher than those of the linear surrogate but this does not mean that the value of H helps us to distinguish between the original time series and their surrogates, because H for the linear surrogate of the first part, NOK, is higher than H for the second part, EUR. For the shuffled surrogate time series we obtain, as expected, Hurst exponents close to 1/2.

As we have seen from the discussion so far, estimators which describe the decay of correlations in a real world process, such as the Nordic electricity spot market can vary quite substantially. This may limit the accuracy and

NOK	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10	
0.44	0.36	0.38	0.34	0.36	0.33	0.42	0.35	0.35	0.31	0.30	(G)
	0.49	0.48	0.48	0.48	0.49	0.50	0.49	0.50	0.49	0.48	(S)
EUR	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10	
0.27	0.12	0.09	0.14	0.16	0.16	0.13	0.09	0.23	0.11	0.17	(G)
	0.43	0.43	0.44	0.41	0.43	0.44	0.43	0.42	0.44	0.43	(S)

Table 1
Hurst exponent for both parts of the Nord Pool time series compared with 10 sets of surrogate data (G) obtained by a Gaussian linear process with the same mean, variance, and power spectrum as the real data, and surrogate data (S) obtained from a shuffled time series.

the interpretation of those results. One way to resolve this dilemma is to characterise certain trends of the Hurst exponent as some control parameters are changed, rather than estimating a single value. Since for the electricity spot market there are no controllable parameters, an 'educated' resampling of the given time series is a sensible way to identify trends in the correlation decay. In particular, we looked at the system price at a certain fixed hour of each day from 1am to 12pm. Such a resampling results in 24 different time series of smaller size. Figure 3(a) shows the power spectrum for the system price at 1am of each day where no distinct peaks can be identified. On the other hand, Fig. 3(b) shows the power spectrum for 8am where distinct peaks can be seen which correspond to the weekly periodicity of the system price. Indeed, this suggests that the system price during night hours is not affected by the 7-day interval of our industrial society, whereas there are strong correlations during daily working hours. This behaviour is summarised in Fig. 3(c) where all 24 power spectra are shown in a three-dimensional representation.

The Hurst exponents estimated from these power spectra are shown in Fig. 4 where different methods for the computation of the exponents have been compared. While the different methods yield quite distinct numerical values all methods essentially produce dips at around 9am and 6pm indicating that at these times the correlations in the system price are strongly dominated by the 7-day interval imposed on the market. Hurst exponents estimated from the asymptotic decay of the correlation function (diamonds) are practically constant although we expect such a method to be the least reliable one. The R/S-method (triangles) and the MF-DFA-method (circles) give practically constant results for the Hurst exponents as well, although with a quite different value. These features might be attributed to the intrinsic averaging of the respective methods. Hurst exponents evaluated from the power spectrum (crosses) and those obtained by the DMA-method (squares) display clearly the daytime dependence. It should be noted that all methods give different results for the estimated Hurst exponent when applied to a real-world time

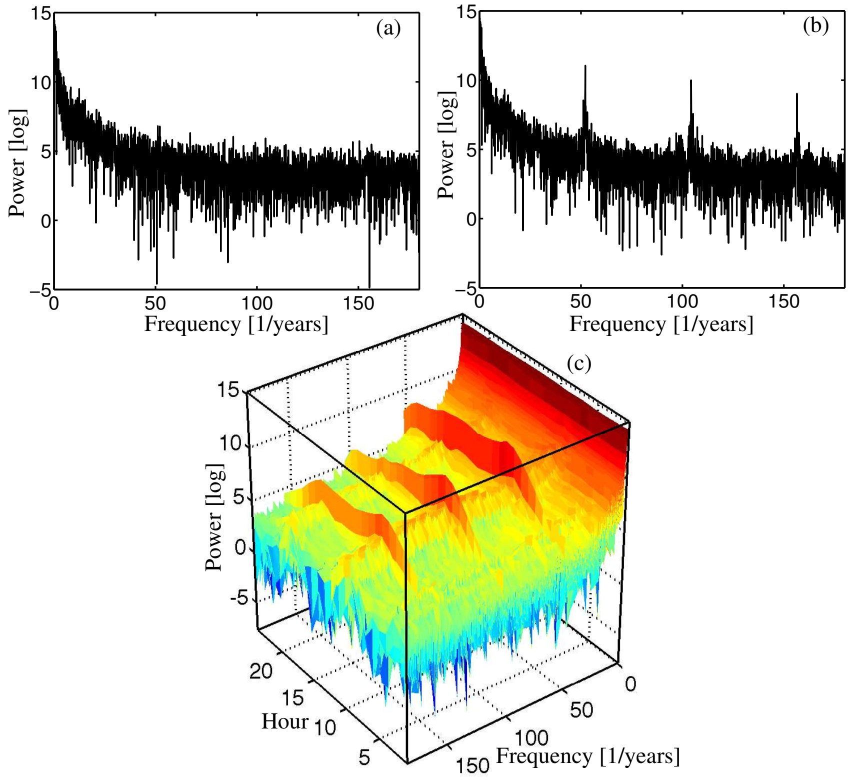

Fig. 3. Weekly periodicities in the system price along different times of the day. Panel (a): Spectrum of the system price at 1am of each day, panel (b): spectrum at 8am of each day, panel (c): three-dimensional representation of the power spectrum for each hour of the day.

series, whereas they give identical results when applied to an ideal self-affine process. Nevertheless, all methods show a dependence on daytime, although at different scales.

5 Conclusion

A perfectly self-affine process can be characterised completely by a single Hurst exponent. However, such a mathematical property is rarely shared by a real world time series. It is therefore sensible to apply more sophisticated data analysis tools, one of which is the generalised multifractal detrended fluctuation analysis, as introduced by [37]. Our analysis of the electricity system price of the Nordic spot market has shown considerable variations of the Hurst exponent, although the results are consistent with a mainly anti-persistent time series as shown by the traditional R/S-method applied to the original time series and to Gaussian linear surrogates with the same mean, variance, and power spectrum. Anti-persistence is preserved while a shuffled time series yields Hurst exponents close to 1/2.

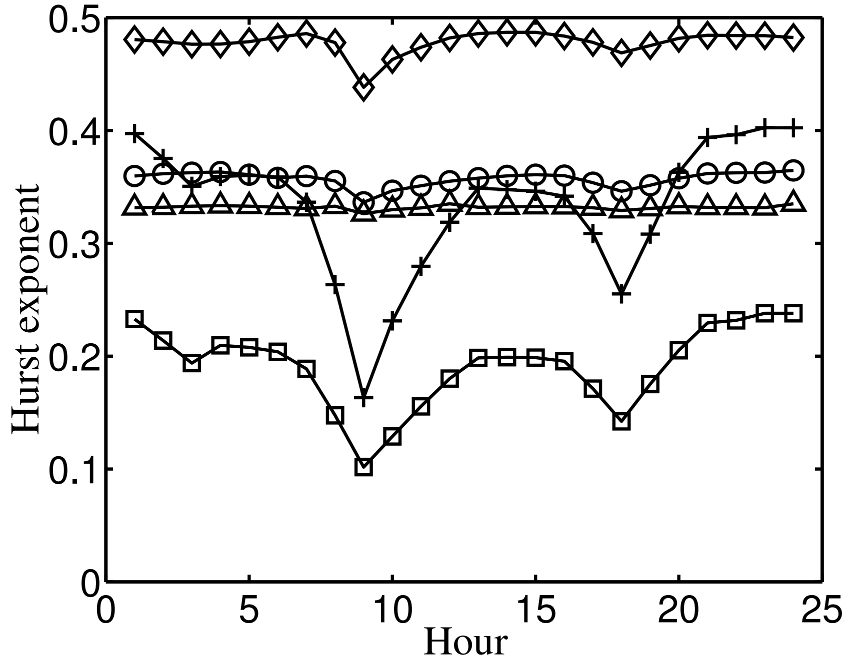

Fig. 4. Evolution of the Hurst exponent during the day estimated from the different spectra obtained from the time series and from the autocorrelation function. Crosses (+): power law scaling of the power spectrum, diamonds (⋄): evaluation of the decay of the correlation function, triangles (△): R/S method, squares (□): DMA, and circles (∘): MF-DFA with first-order interpolation of the time series.

To illustrate the large fluctuation properties of the Nord Pool data more clearly we may compute a time-depended Hurst exponent as well. To this end we resample the complete time series in overlapping time windows of different length (1,000 hours , 5,000 hours, and 10,000 hours) and use the power spectrum to estimate the Hurst exponent of the respective time window. On the one hand this allows to estimate the fluctuations of the Hurst exponent on different time scales. On the other hand this gives an estimation for the accuracy of the Hurst exponent when only a finite number of data points is available. Figure 5 shows the results for the system price. Large fluctuations of the Hurst exponent can be seen when computed from windows of size 1,000 hours. Fluctuations become smaller as the length of the time window increases, however, even for time windows of 10,000 hours fluctuations are still substantial. This indicates that the actual value of H may vary strongly, depending on the time for which it is estimated. Only time windows of more than 100,000 hours give a practically constant Hurst exponent similar to the value obtained in section 3. In other words, for the estimation of the Hurst exponent or any other quantitative measure of the correlation decay the finite length of the time series may have important consequence on the outcome.

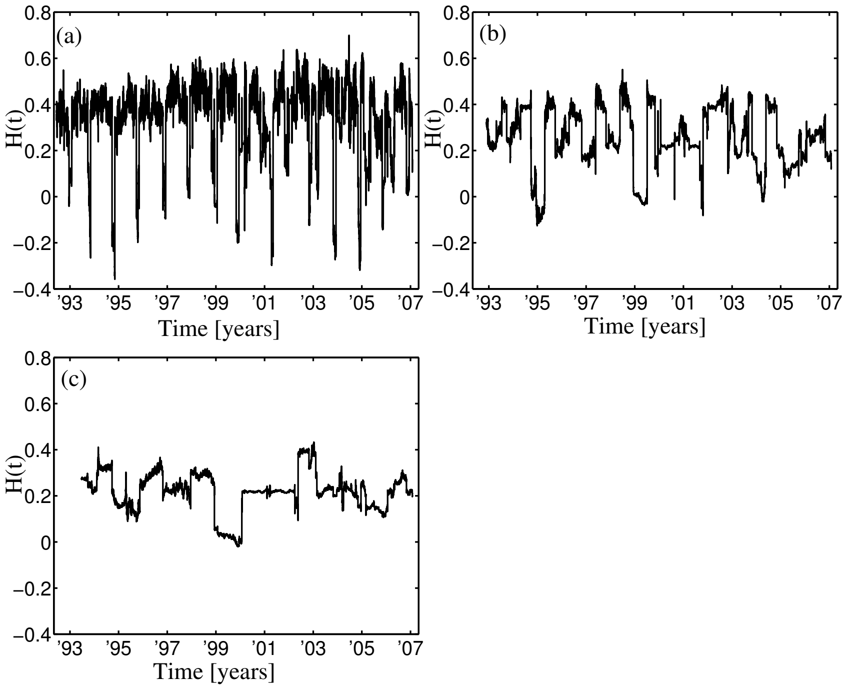

Fig. 5. Time fluctuations of the Hurst exponent for different sizes of a moving window obtained from the decay in the power spectrum. Window sizes are (a) 1,000 hours, (b) 5,000 hours, and (c) 10,000 hours.

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References

[1] R. N. Mantegna and H. E. Stanley, *An Introduction to Econophysics*, Cambridge Univ. Press. (2000).

[2] N. F. Johnson, P. Jefferies, and P. Ming Hui, *Financial Market Complexity*, Oxford Univ. Press. (2003).

[3] M. F. M. Osborne, *Brownian motion in the stock market*, Oper. Res. 7 (1959) 145.

[4] B. Malkiel, *A Random Walk Down Wall Street*, Norton, New York (1990).

[5] B. B. Mandelbrot, *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk*, Springer, New York (1998).

[6] W. A. Brock, D. A. Hsieh, and B. LeBaron, *Nonlinear Dynamics, Chaos and Instability: Statistical Theory and Economic Evidence*, MIT Press, Massachusetts, MA (1991).

[7] D. A. Hsieh, *Chaos and nonlinear dynamics: Application to financial markets*, J. Finance 46 (1991) 1839.

[8] H. W. Lorenz, *Nonlinear Dynamical Economics and Chaotic Motion*, Springer, New York (1993).

[9] E. E. Peters, *Chaos and Order in the Capital Markets: a New View of Cycles*, Prices and Volatility, 2nd Edition, Wiley, New York (1996).

[10] R. N. Mantega and H. E. Stanley, *Scaling behaviour in the dynamics of an economic index*, Nature 376 (1995) 46.

[11] R. N. Mantegna and H. E. Stanley, *Turbulence and financial markets*, Nature 383 (1996) 587.

[12] P. Bak and K. Chen, *Self-organized criticality*, Sci. Amer. 264 (1991) 26.

[13] M. F. Shlesinger,G. M. Zaslavsky, and J. Klafter, *Strange kinetics*, Nature 363 (1993) 31.

[14] J. Scheinkman and B. LeBaron, *Nonlinear dynamics and stock returns*, J. Business 62 (1989) 311.

[15] G. Papaioannou and A. Karytinos, *Nonlinear time series analysis of the stock exchange: The case of an emerging market*, Int. J. of Bif. and Chaos 5 (1995) 1557.

[16] I. Andreadis, *Self-criticality and stochasticity of an S&P 500 index time series*, Chaos, Solitons and Fractals 11 (2000) 1047.

[17] R. Friederich, J. Peinke, and C. Renner, *How to quantify deterministic and random influences on the statistics of the foreign exchange market*, Phys. Rev. Lett. 84 (2000) 5224.

[18] A. S. Soofi and L. Cao, *Modelling and Forecasting Financial Data: Techniques of Nonlinear Dynamics*, Kluwer Academic Publishers, Norwell (2002).

[19] R. Weron and B. Przybylowicz, *Hurst analysis of electricity price dynamics*, Physica A 283 (2000) 462.

[20] I. Simonsen, *Measuring anti-correlations in the Nordic electricity spot market by wavelets*, Physica A 322 (2003) 597.

[21] H. N. E. Byström, *Extreme value theory and extremely large electricity price changes*, Int. Rev. Econ. and Finance 14 (2005) 41.

[22] J. Perelló, M. Montero, L. Palatella, I. Simonsen, and J. Masoliver, *Entropy of the Nordic electricity market: anomalous scaling, spikes, and mean-reversion*, J. Stat. Mech., 2006 (2006) P11011.

[23] R. Weron, M. Bierbrauer, and S. Truck, *Modelling electricity prices: Jump diffusion and regime switching*, Physica A 336 (2004) 39.

[24] I. Vehviläinen and T. Pyykkönen, *Stochastic factor model for electricity spot price-the case of the Nordic market*, Energy Economics 27 (2005) 351.

[25] I. Simonsen, *Volatility of power markets*, Physica A 355 (2005) 10.

[26] R. Weron, *Energy price risk management*, Physica A 285 (2000) 127.

[27] P. Norouzzadeh, W. Dullaert, B. and Rahmani, *Anti-correlation and multifractal features of Spain electricity spot market*, Physica A 380 (2007) 333.

[28] A. Serletis and A. A. Rosenberg, *The Hurst exponent in energy futures prices*, Physica A 380 (2007) 325.

[29] A. Provenzale, L. A. Smith, R. Vio, and G. Murante, *Distinguishing between low-dimensional dynamics and randomness in measured time series*, Physica D 58 (1992) 31.

[30] H. Kantz and T. Schreiber, *Nonlinear Time Series Analysis*, Cambridge Univ. Press (1997).

[31] F. Strozzi, J. M. Zaldívar, and J. P. Zbilut, *Application of nonlinear time series analysis techniques to high frequency currency exchange data*, Physica A 312 (2002) 520.

[32] T. Gneiting and M. Schlather, *Stochastic models that separate fractal dimension and the Hurst effect*, SIAM Review 46 (2004) 269.

[33] H. E. Hurst, *Long-term storage capacity of reservoirs*, Trans. Am. Soc. Civ. Eng. 116 (1951) 770.

[34] B. B. Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman, New York (1983).

[35] E. Alessio, A. Carbone, G. Castelli, and V. Frappietro, *Second-order moving average and scaling of stochastic time series*, Eur. Phys. J. B 27 (2002) 197.

[36] A. Carbone, G. Castelli, and H. Stanley, *Time-dependent Hurst exponent in financial time series*, Physica A 344 (2004) 267.

[37] C.-K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, and A. L. Goldberger, *Mosaic organization of dna nucleotides*, Phys. Rev. E 49 (1994) 1685.

[38] A. Bunde, S. Havlin, J. W. Kantelhardt, T. Penzel, J.-H. Peter, and K. Voigt, *Correlated and uncorrelated regions in heart-rate fluctuations during sleep*, Phys. Rev. Lett. 85 (2000) 3736.

[39] M. J. Cannon, D. B. Percival, D. C. Caccia, G. M. Raymond, and J. B. Bassingthwaighte, *Evaluating scaled windowed variance methods for estimating the Hurst coefficient of time series*, Physica A 241 (1997) 606.

[40] J. W. Kantelhardt, S. A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, and H. E. Stanley, *Multifractal detrended fluctuation analysis of nonstationary time series*, Physica A 316 (2000) 87.