A supply chain as a series of filters or amplifiers of the bullwhip effect

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Abstract

The bullwhip effect refers to the phenomenon of amplification and distortion of demand in a supply chain. By eliminating or controlling this effect, it is possible to increase product profitability reducing useless costs such as stock-out and obsolescence costs. The main focus of this work is to study a single-product serial supply chain in which a control parameter can switch the chain from a series of filters to a series of amplifiers of the bullwhip effect and to analyse how the optimal values of the parameters change when discontinuities in order policy are considered. Furthermore, it is also shown that the bullwhip itself it is not a good index of the chain's performance, because it does not consider the oscillations that occur in the inventories, which also may affect the supply chain performance.

Keywords. Bullwhip effect, supply chain, order policy.

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1. Introduction

The bullwhip effect refers to a phenomenon that occurs in the supply chains when orders to the supplier have a larger variance than the ones from the customers, i.e. demand distortion. This distortion propagates upstream in an amplified form, i.e. variance amplification (Geary et al., 2006).

The first academic description of the bullwhip phenomenon is usually ascribed to Forrester (1961), who explained it as a lack of information exchange between the components of the supply chain and by the existence of non-linear interactions, which were difficult to deal with using managerial intuition. In recent years, several models have been developed for examining different factors that may have an impact on this effect. Metters (1997) tried to identify the magnitude of the problem by establishing an empirical lower bound of the bullwhip effect on profitability. Metters (1997) showed that by eliminating the seasonal bullwhip effect alone, one can increase the product profitability by 10-20%, while decreasing the bullwhip effect due to forecast error it was possible to increase profits by 5-10%. The combined profits by removing seasonality and forecast error, produced an increase in profits around 15-30%. In Chen et al. (2000) the bullwhip effect for a simple supply chain consisting of a single retailer and a single manufacturer was quantified. It was assumed that there is a correlation between the actual demand and its past values while the retailer applies an order policy based only on past demand. Using these assumptions, the impact of forecasting, lead time and information on the bullwhip effect was measured when the variance of the demand increases. Furthermore, Chen et al. (2000) found that, providing customer data information to every stage of the supply chain, the magnitude of the bullwhip effect can be decreased, but still exists when the demand information in each stage is centralized. With the order policy considered, the bullwhip is always bigger than one. The beneficial effects of information sharing and quality of that information in supply chains were examined for reducing the bullwhip effect by Dejonckheere et al. (2004) and Chatfield et al. (2004).

Following a different approach, Burns and Sivazlian (1979) described a supply chain as a sequence of amplifiers in the frequency domain using the z-transform, whereas Towill et al. (1994 and 2003) developed a similar approach using the Laplace transform and filtering the disturbances in the demand signal producing a supply chain robust with respect to random variation in the demand. Following a systems analysis approach, Chen and Disney (2003) were able of reducing the bullwhip effect by controlling the order policy by using a proportional controller. An improvement in the cost saving due to the reduction of order variance was obtained. Soft computing methods were applied by Carlsson and Fullér (2001) to reduce the bullwhip effect. A policy in which orders were imprecise was applied. Orders were considered as intervals and the actors in the supply chain had to make their orders more precise as the time point of delivery got closer. In that policy crisp orders were replaced by fuzzy numbers. The problem was that the fuzzy system itself was not able learn the membership function therefore a neural network was used to approximate the crisp value. Also in this case, the bullwhip was significantly reduced (Carlsson and Fullér, 2001).

The supply chain was also modelled using Petri Nets (Makajic-Nikolic et al, 2004). The authors considered a supply chain as a business process which had to be redesigned assuming that the main cause of the bullwhip effect was the absence of coordination in the management of the supply chain. Petri Nets were used as a simulation tool to support a decision-maker in choosing the best-fit scenario and in increasing the coordination. Genetic algorithms (GA) have also been developed to optimize the base-stock levels and reduce the bullwhip effect with the final objective of minimizing the sum of holding and shortage costs in the entire serial single-product supply chain (Disney et al., 2000; Sudhir and Chandrasekharan, 2005; Strozzi et al., 2007) . The robustness of this approach with respect to changes in the supply line (Sudhir and Chandrasekharan, 2005) as well as in the customer demands (Strozzi et al., 2007) was also assessed.

The objectives of this work are to show how a simple and realistic order policy can reduce or amplify the bullwhip effect and the inventory oscillations in a serial single-product four echelons supply chain, and also to analyse the impact of discontinuities of this order policy on the bullwhip and maximum oscillation surfaces. Moreover, we have also analysed how this order policy may be optimized to reduce the bullwhip and oscillations in the inventories under different customer demands with and without noise.

2. Supply chain Model

In this work we consider a serial single-product distribution system of four levels similar to the one presented by Sterman (1989) and Mosekilde et al. (1991) in which the actors are the Factory, Distributor, Wholesaler and Retailer, see Figure 1. The Customer asks for the goods from to the Retailer which, in turns, asks for goods from the Wholesaler and so on until the orders reach the Factory. In the mean time the goods are going from the factory down through the chain until they reach the Customer.

[Figure 1]

In order to simplify this production-distribution system several rules were defined by Sterman (1989) and Mosekilde et al. (1991): there is only one inventory at each level; the time delay from passing of orders and shipments from one level to the next is fixed to one week (one time period); the production time is taken to be two weeks, and it is assumed that the production capacity of the factory has no limit; each week Customer orders goods from the Retailer, who supplies the requested quantity out of inventory.

The simulation model consists of a high-dimensional iterated map that provides the sequence of operations that each sector should perform. The boxes in Fig. 1 represent the state variables. Each variable has a letter that indicates the respective sector; thus *R* stands for retailer, *W* for wholesaler, *D* for distributor and *F* for factory. For example, in the wholesaler sector, *WINV* is the wholesaler inventory, *WBL* the wholesaler backlog of orders, *WIS* and *WOS* represent incoming and outcoming shipments, respectively, where *WIO* is the incoming orders, *WED* is the expected demand and *WOP* the orders placed by the wholesaler. One time step later, *WOP* becomes incoming orders to the distributor, *DIO*. The same notation is employed in the other sectors with the exception of the factory where there is a production rate, *FPR*, instead of placed orders and where *FPD* represents the production delay. The exogenous customer order rate is depicted by *COR*.

The difference equations of the model that represent the operations conducted in each echelon may be written for the wholesaler as follows.

$$WINV_t = \begin{cases} WINV_{t-1} + WIS_t - WBL_{t-1} - WIO_t & if \quad WINV_{t-1} + WIS_t \geq WBL_{t-1} + WIO_t \\ 0 \quad otherwise \end{cases} \tag{1}$$

$$WIS_t = DOS_{t-1} \tag{2}$$

$$WBL_t = \begin{cases} WBL_{t-1} + WIO_t - WINV_{t-1} - WIS_t & \text{if } WBL_{t-1} + WIO_t \geq WINV_{t-1} + WIS_t \\ 0 \quad \text{otherwise} \end{cases} \tag{3}$$

$$WIO_t = ROP_{t-1} \tag{4}$$

$$WOS_t = \min\{WINV_{t-1} + WIS_t, WBL_{t-1} + WIO_t\} \tag{5}$$

Concerning the remaining sectors: *RINV*, *DINV* and *FINV* have similar expressions as in Eq. (1); *RIS*, *DIS* and *FIS* are written as Eq. (2); similar expressions as in Eq. (3) are written for *RBL*, *DBL* and *FBL*; whereas Eq. (4) holds for *DIO* and *FIO*; finally, for *DOS*, *FOS* and shipments out of retailer's inventory, Eq. (5) is applied.

3. The Order Policy

The orders at the end of week *t-1* arrive at the beginning of week *t* at the upper level. In the mean time the actor decides upon the outgoing shipments at week *t* and the goods will arrive at the beginning of week (*t+1*) at the upper level. The orders can be fulfilled after two weeks. In order to have the same delay in the Factory we add the box *FPD* then the orders at week *t-1* will go in the box *FPD* at week *t* and in the incoming shipments, *FIS*, in the week *t+1*, i.e.

$$FPD_t = FPR_{t-1} \text{ and } FIS_t = FPD_{t-1} \qquad (6)$$

For forecasting the expected demand we use an exponential smoothing as did Sterman (1989), thus

$$WED_t = T \cdot WIO_t + (1-T) \cdot WED_{t-1} \qquad (7)$$

WED_t and WED_{t-1} are the expected demand at times *t* and *t-1*, respectively, WIO_t is the incoming orders, and T $(0 \le T < 2)$ is a parameter that controls the rate at which expectations are updated. $T = 0$ corresponds to stationary expectations, and $T = 1$ describes a situation in which the immediately preceding value of received orders is used as an estimate of future demand. $T = 0$ means that the actor uses the long term average demand as a forecast, WIO_t, whereas $T=1$ means that the actor simply passes on demand. That is the forecast in the last observed demand. The values of T between one and two $(1 < T < 2)$ represent the situation when changes in demand are overreacted too by exponential smoothing.

The actors place orders by considering several variables. If we consider the stock adjustment, the incoming shipment, the orders placed and order that were not fulfilled at time *t-1*, the order policy of one echelon, for example wholesaler, will be:

$$WOP_t = \max(0, WED_t + (Q - WINV_t + WBL_t) - (WOP_{t-1} + DBL_{t-1})) \qquad (8)$$

Wholesaler sector makes orders in accordance with the expected demand, WED_t, adding the quantity missing to reach the desired stock level, *Q*, subtracting the goods already ordered in the week *(t-1)* and that were not provided and the orders already placed. The function *max* is used to avoid negative orders.

Applying this order policy to a step function customer demand, the effective inventory will always converge (see Fig. 2) independently from the initial conditions (backlog and inventory at time zero). The time to reach the equilibrium depends upon *T*.

The Wholesaler orders exactly the quantity that is necessary to adjust the stock considering only the incoming shipments and the expected demand. When the orders reach the incoming shipments Wholesaler stops ordering and the desired level, *Q* = 14, is never reached, because shipments stop circulating and the desired inventory cannot be maintained to the desired level.

[Figure 2]

In order to reach and maintain the desired inventory, the wholesaler has to take in account the supply chain and to order some more goods, which move along in the chain and guarantee continuous availability of the stock.

We may try to correct this model, even when the equilibrium is reached, by taking in account the goods that we want circulating at time *t* that we will call Supply Line at time t (SL_t). Since we do not know exactly what will be the customer demand we suppose that SL_t will be the same that expected demand: $SL_t = WED_t$. With this approach it is possible to define a new order policy as:

$$WOP_t = \max(0,\ 2 \cdot WED_t + (Q - WINV_t + WBL_t) - (WOP_{t-1} + DBL_{t-1})) \qquad (9)$$

As one can see in Fig 3, with this policy, the effective inventory converges to the desired inventory Q = 14.

[Figure 3]

In this case, high values of T would produce high oscillations before reaching the equilibrium whereas low values of T would increase the time necessary to reach equilibrium. This policy leads to the convergence of the stock if the customer demand converges. If the customer demand fluctuates around an equilibrium value, then the stock will fluctuate around another equilibrium value.

Now we want to modify the order policy with the purpose of giving different weights to the short and long term demand forecast, in order to analyse different behaviours provided by this policy. To this end, let us decompose WED_t in two parts: the first, $WEDS_t$, represents the short term customer demand forecast, while the second, $WEDL_t$, represents the long term customer demand forecast and also indicates the quantity that we want to circulate (SL). It is possible then to write:

$$WOP_t = \max(0, WEDL_t + \alpha \cdot [WEDS_t + (Q - WINV_t + WBL_t) - (WOP_{t-1} + DBL_{t-1})]) \qquad (10)$$

Good choices for $WEDS_t$ and $WEDL_t$, in order to preserve the convergence, are respectively: the last order WIO_t, and the exponential smoothing $T \cdot WIO_t + (1-T) \cdot WEDL_{t-1}$. The parameter α, that we suppose varies between 0 and 2, is the weight given by an agent of the chain to the history of the demand and, at the same time, to the quantity of goods that he wants in the supply chain. Therefore we will be considering:

$$WEDS_t = WIO_t \tag{11}$$

$$WEDL_t = T \cdot WIO_t + (1-T) \cdot WEDL_{t-1} \tag{12}$$

Despite its simplicity, this model succeeds in maintaining the effective inventory (*INV-BL*) to a desired level, Q=14 (Fig 3) and its order policy is easier to apply since less information is required when compared with the model in Sterman (1989).

4. The supply chain as a series of filters or amplifiers

Figures 4-7 shown the effective inventories (inventories minus backlogs) and the orders placed, applying the defined order policy (Eqs. 10-12), when the customer demand is a step function, a stairs function, a sine function or a function correlated with its past values (Table 1):

$$COR_t = \rho \cdot COR_{t-1} + m + \varepsilon_t \tag{13}$$

where ρ is the correlation factor, *m* is a constant to avoid negative orders and ε_t is a noise term with mean 0 and variance σ. The cases have been analysed with and without noise added to the demand.

[Figures 4-7]

In Figures 4-7, it is possible to observe how the order policy reduces the oscillations in going from the retailer to the factory. Moreover, by analyzing the customer demands affected by noise, it is possible to see that the oscillations are filtered when they reach the factory. Of course they are shifted in time too due to the delays in sending and receiving the orders. By increasing α, i.e. giving more weight to the short term forecast, the logistic chain becomes an amplifier of the customer demand as we can see in Fig. 8 for the case of $\alpha = 0.6$.

[Figure 8]

It is not the first time in the literature that this effect has been observed in logistic chains. For example, they were seen as series of filters (Towill and Del Vecchio, 1994) or as a series of amplifiers (Burns and Sivazlian, 1979) considering different order policies. However, here we have shown that a unique order policy exists that is able to transform the logistic chain from a series of filters to a series of amplifiers and *viceversa*. Therefore, even if the purpose of having a supply chain – and the stocks at different levels- is in principle, to reduce oscillations in the customer demands, it is possible to see that a variation of the parameter α could transform a well behaved supply chain in a series of demand amplifiers.

5. Bullwhip effect

The amplification of the oscillations can be measured using the bullwhip index (Chen et al., 2000), *b*, defined as the quotient between the variance of the incoming order into the factory, *var(FIO)*, and the variance of the customer demand, *var(COR).*

In Fig. 9 the bullwhip index, *b*, is represented as a function of the parameters α and *T* with a stair and a sinusoidal function representing the customer demand with and without noise. The surface corresponding to *b*=1 is also shown. This surface represents the case in which the variance of the customer demand does not change. The bullwhip surface has a minimum in correspondence of low values of α (the weight given to the short term customer demand) and low values of *T* (rate of reaction to new orders). The minimum of the bullwhip surface corresponds to *b* values smaller than one. The same effect happens for other customer demands considered in Figs. 5, 7 and 8 (figures not shown). Bullwhip smaller than one means that the variance of the customer is higher than the variance of the factory and, therefore, the chain is acting as a filter. The optimal policy, looking to the bullwhip, seems to behave with a damped response when confronted with sudden changes in the customer requests and do not take into account short term forecast.

[Figure 9]

However, it seems obvious that this cannot be a good order policy. In fact, for low values of α and *T*, the low oscillations of the incoming orders into the factory are paid for by wide oscillations in the inventories as it is possible to see in Fig 10, where for each echelon the maximum difference during a simulation run between [*max*(*INV-BL*)-*min*(*INV-BL*)] is represented for several customer demands. This value complements the information provided by the bullwhip index since it is possible to have *b* values far from one, but wide oscillations in the inventory.

[Figure 10]

The surfaces for the other customer demands like those of Figs 5, 7, 8 (not shown) are similar to the one provided by the step function. In Figure 10 one can see that the maximum oscillations, decrease with *T* only when α is not too small (α nearly 0.5) i.e. if the actors consider the adjustment of supply chain and stock. If we add noise to the demand the surface does not change qualitatively, i.e. the minimum, again, is for α values between 0.4 and 0.5 and for low values of *T*. These properties do not depend from the number of weeks considered.

Looking to the surfaces generated in Figs. 9-10, it is possible to observe that the optimal values of *T* and α are the ones that provide a bullwhip close to 1 and minimize the inventories oscillations. The maximum in the oscillations always decreases with *T*. As a conclusion, the optimal values will be *T* minimum (in the simulation considered the minimum value is 0.02) and α between 0.3 and 0.5, see Table 2 for the optimal values. In the limit, when *T*=0, *WEDL* is a constant:

$$WOP_t = const + \alpha \cdot \left[WIO_t + (Q - WINV_t + WBL_t) - (WOP_{t-1} + DBL_{t-1}) \right] \tag{14}$$

Therefore, *T* = 0 means that the incoming order is not taken into account in the order forecast and then the supply chain will never reach the convergence to the desired level of inventory. This order policy will be similar to the one given by Eq. (8).

- *Independence of bullwhip from Q parameter*

For all the customer demands of Table 2 and order policy given by Eqs. (10)-(12), we have verified that the maxima oscillation and the bullwhip are independent from the *Q* parameter. As an example, Fig. 11 shows the bullwhip for COR_3, COR_4, COR_5, COR_6, when *Q* and *T* parameters change.

[Figure 11]

For this reason we did not consider the variation of the parameter *Q* in the former analysis of the optimal parameter policy.

6. The impacts of order policy discontinuity

In order to analyse the situation in which negative orders occur maintaining the shape of customer demands defined in Table 2, we have subtracted some units to the customer orders. With a shift of four units and α=0.3, we obtain Figs. 12-13.

[Figure 12-13]

If we change α to 0.6 we also obtain negative orders for the same customer demands (Figs 14-15) but the Factory incoming orders have higher oscillations. In both cases, α=0.3 and α=0.6, it is possible to observe, mainly for COR_5 and COR_6, that the effective inventory (INV-BL) of the retailer absorbs all the fluctuations due to wrong demand forecast and this avoids the propagation of the oscillations along the chain.

[Figures 14-15]

- *Bullwhip Surfaces*

The bullwhip surfaces of our order policy corresponding to customer demands COR_3, COR_4, COR_5 and COR_6 are represented in Fig 16. Negative orders shift the bullwhip surface bellow the constant surface b=1 for COR_5 and COR_6, as can be seen when comparing with Fig. 9, whereas small differences can be observed for COR_3, COR_4.

[Figure 16]

- *Maximum differences surfaces*

Negative orders increase the maximum differences surfaces (see Fig. 17) compared with the case in which negative orders are not generated (see Fig. 10). This phenomenon becomes more evident in presence of noise.

[Figure 17]

- *Bullwhip as a function of T and Q*

Plotting bullwhip surface in correspondence of different values of T and Q parameters (Figs. 11 and 18) one can see that the independence from Q values i.e. from desired inventory (supposed the same for each echelon) does not change. This happen for all the orders considered but we plot them only for the customer demands for which we would have negative orders to the factory (COR_3, COR_4, COR_5, COR_6). The surfaces correspondent to negative orders reach maximum values smaller than the ones correspondent to positive orders (Fig. 11) and they amplify noise. In two cases (COR_5, COR_6) they have completely different behaviour: the bullwhip surfaces have an opposite inclination even if very small (they are nearly flat).

[Figure 18]

In order to analyse the impacts of negative orders on the optimal values of α, we have translated customer orders from 0 to 14 units. Optimal α values are the ones that maintain the stability of the supply chain: in the sense that minimise the maximum variations in the inventories (Fig. 19), and maintain the bullwhip close to one (Fig. 20). In order to maintain the stability of the supply chain when negative orders increase, it seems necessary to increase α, as can be observed in Fig. 19 (right column), which means giving more importance to the short term with respect to the long term demand. Figure 20 (right column) points also in the same direction. In both cases, for higher translation any value of α is not able to recover the high differences that are produced in the effective inventories (*INV-BL*), therefore these values are not significative. When the noise is introduced in the demand (Figs. 19-20, left column) the profile of optimal α values increases again, but the behaviour is not so clear.

7. Conclusions and future developments

In this work we have shown an order policy that, applied to the sectors of a serial single-product supply chain with four echelons, can reduce or amplify the bullwhip effect and the inventory oscillations. Despite its simplicity, this model succeeds in maintaining the effective inventory to a desired level and its order policy is easier to apply since less information is required when compared with the Sterman's model (1989).

Depending on the values of the model parameters, the supply chain with the order policy given by Eqs. 10-12 behaves as a filter or as amplifier. We have checked the robustness of this policy in respect to different customer demands. The surfaces of bullwhip and the ones of maximum oscillations of inventories have been obtained as a function of the model parameters: α (weight given to the history of the demand) and T (the importance given to the last incoming order). The bullwhip and the maximum oscillation surfaces have a similar characteristic shape for all demands. A conclusion that immediately emerges by analysing the reported results, is that the bullwhip alone does not give the real value of the global performance of the chain since low bullwhip in the demand can be obtained at the cost of having high oscillations in the inventories as it was already shown by Disney et al. (2003) and (2004). Using both, bullwhip and maximum oscillations surfaces, it is possible to draw conclusions about the optimal parameters for the considered policy (see Table 2). We considered a policy optimal when the oscillations in the inventories are minimal and when the bullwhip values are close to one.

Additionally, we checked the impact on the bullwhip and maximum oscillation surface, of discontinuities in the order policy. Bullwhip surfaces (Figs. 16-18) become smoother and they approach b=1 from below, which means that the Factory Incoming Orders variance is smaller than the Customer order variance. However, these smaller oscillations give rise, as a drawback, to higher oscillations in the effective inventory of the Retailer (Fig. 12), which absorbs all the fluctuations of the customer demand. This can be avoided allowing different order policies (i.e. different optimal parameters values α and T) for each echelon of the supply chain. To analyse the impact of negative orders on optimal α values we shifted the orders of the customer by several units. We observed that, in order to maintain small oscillations in the inventories and bullwhip values near to one, one has to increase α values and therefore increase the importance given to short term demand.

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FIGURES AND TABLES

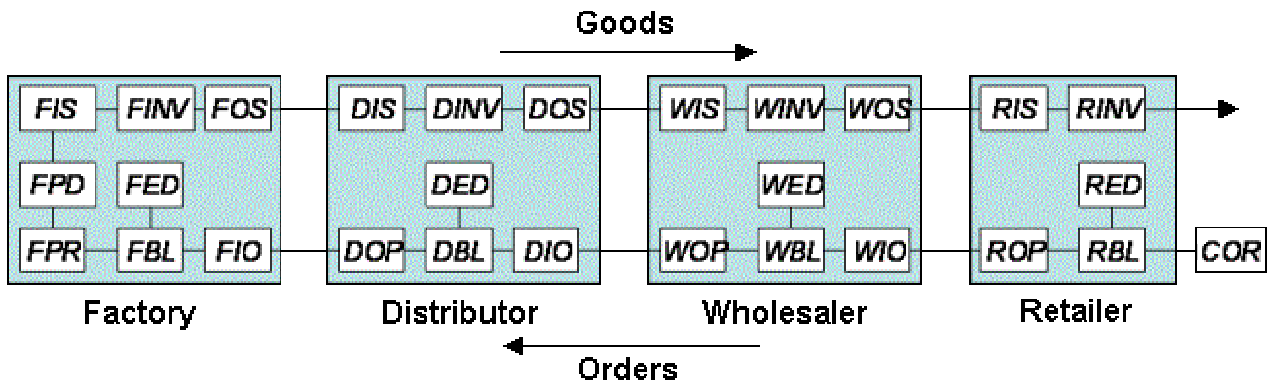

Figure 1. Basic structure of production-distribution system with state variables and orders flow (left arrow) and goods flow (right arrow) in the supply chain model.

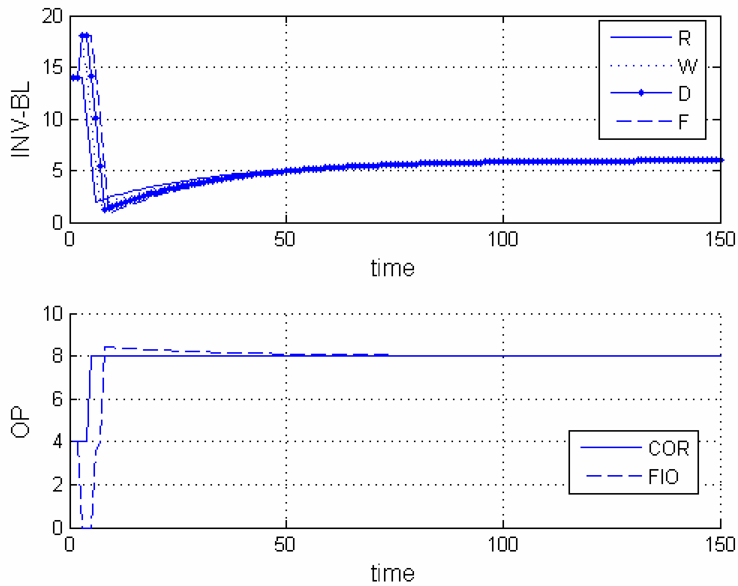

Figure 2. Effective inventory (inventory minus backlog) and orders obtained using policy defined by Eq. (8) with T=0.03 and Q=14, when the customer demand (COR_1) is a step function.

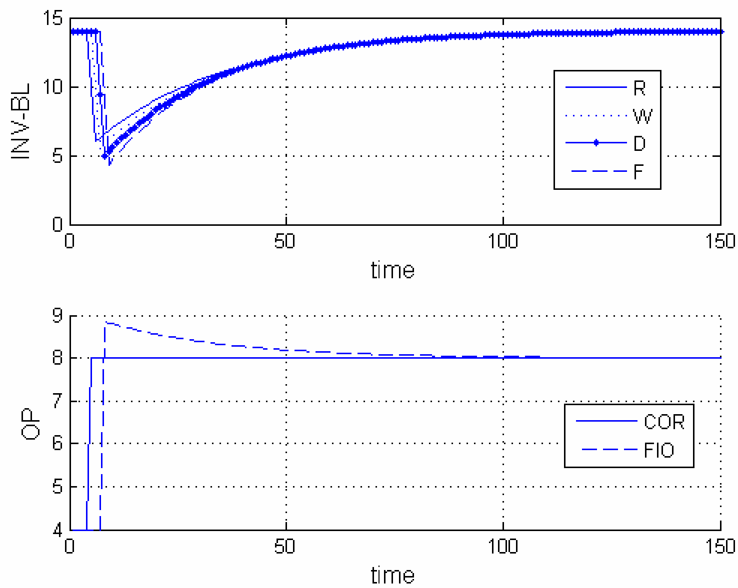

Figure 3. Effective inventory (inventory minus backlog) and orders obtained using policy defined by Eq. (9) with T=0.03 and Q=14, when the customer demand (COR_1) is a step function.

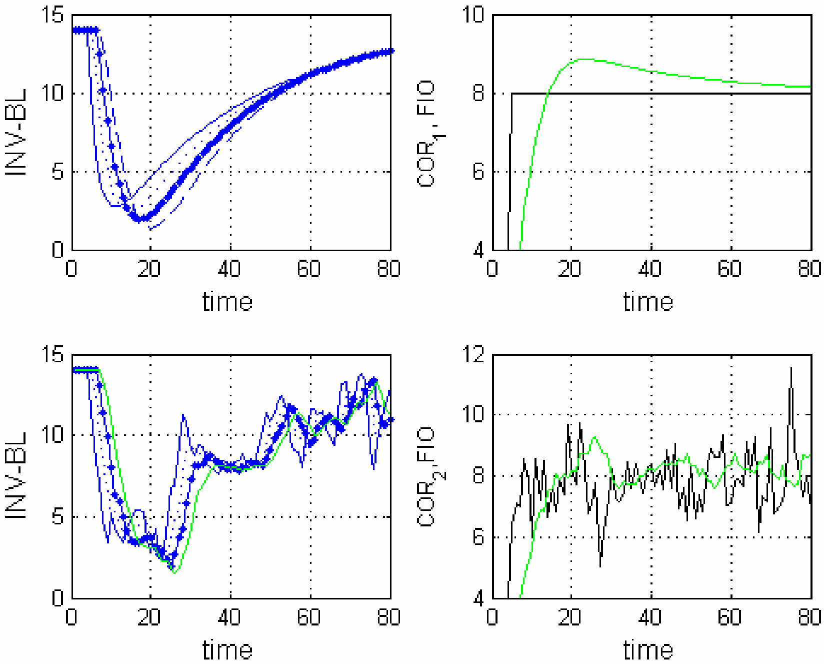

Figure 4. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a step function without (COR_1) and with (COR_2) noise. $\alpha = 0.3$, T=0.03, Q=14 and σ=1.

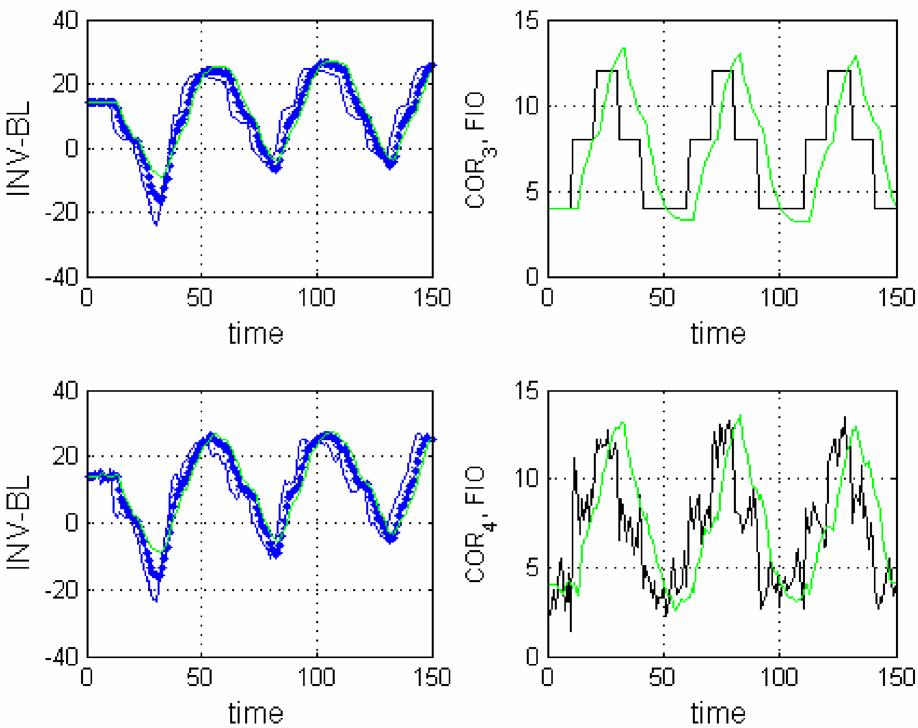

Figure 5. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a stairs function with (COR_3) and without (COR_4) noise. α = 0.3, T = 0.03, Q=14 and $\sigma = 1$.

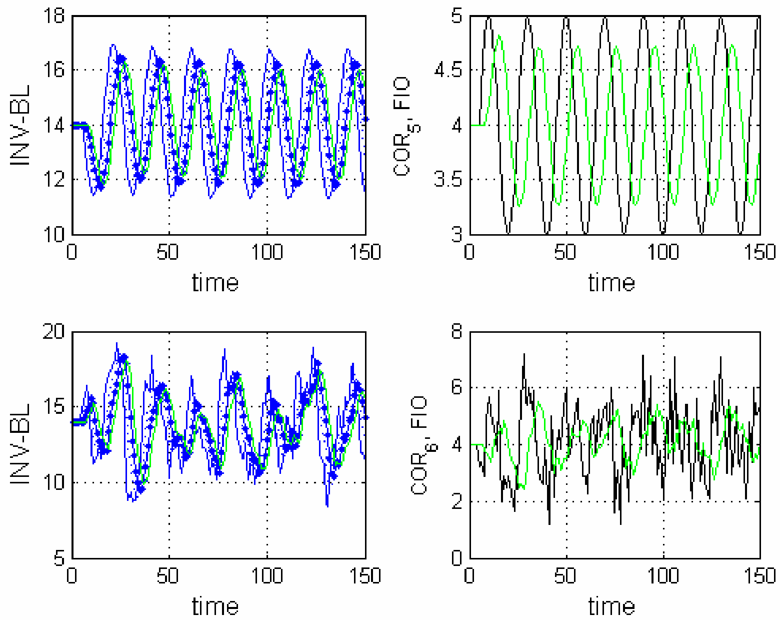

Figure 6. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a sinusoidal function without (COR_5) and with (COR_6) noise. $\alpha = 0.3$, $T = 0.03$, $Q = 14$ and $\sigma = 1$.

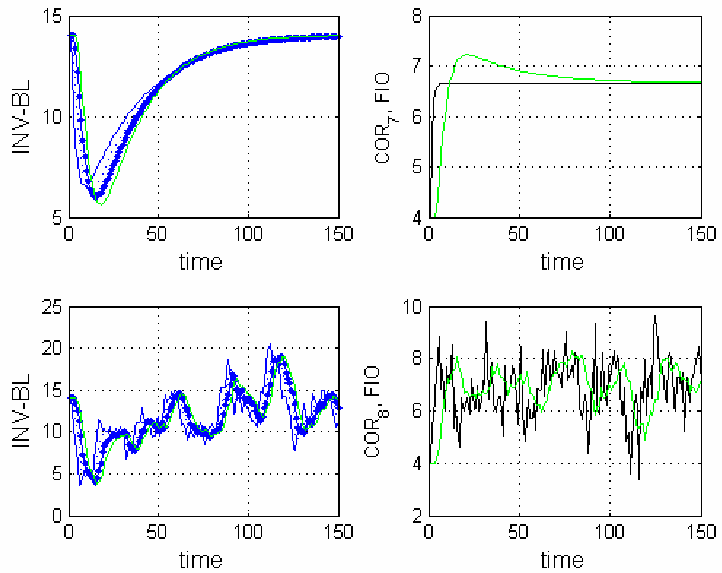

Figure 7. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is the correlated function given by Eq. (13) without (COR_7) and with (COR_8) noise. $\alpha = 0.3$, $T = 0.03$, $Q = 14$, $\rho = 0.4$ and $\sigma = 1$.

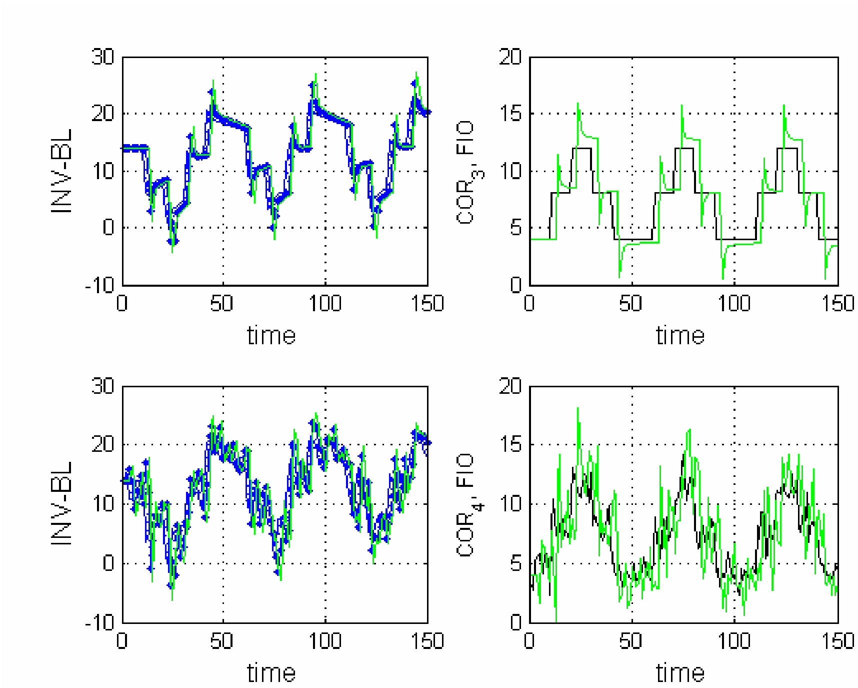

Figure 8. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a stair function without (COR_3) and with (COR_4) noise. $\alpha = 0.6$, $T = 0.03$, $Q = 14$ and $\sigma = 1$.

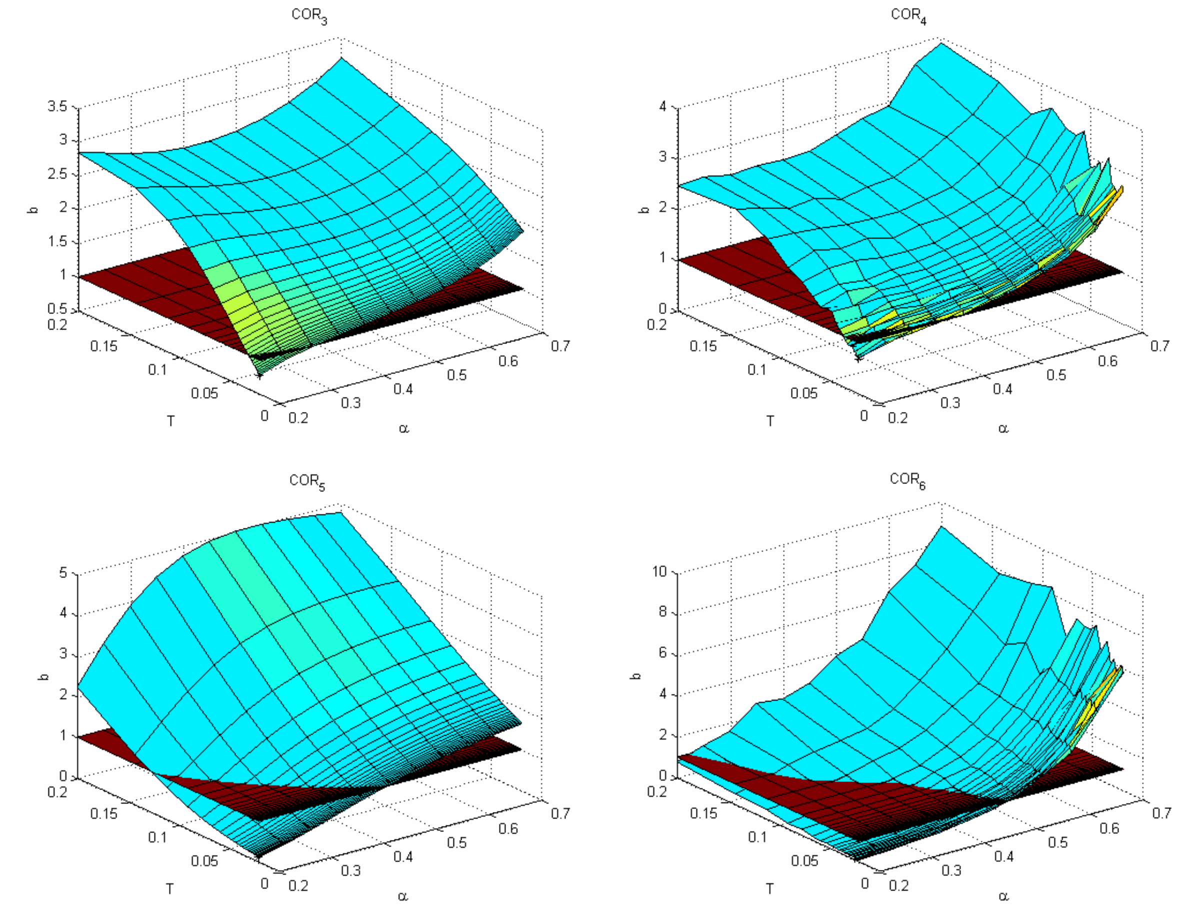

Figure 9. Bullwhip surfaces in (α, T) plane Q=14 and σ=1. Parameters α=[0.2-0.7] and T=[0.02-0.2].

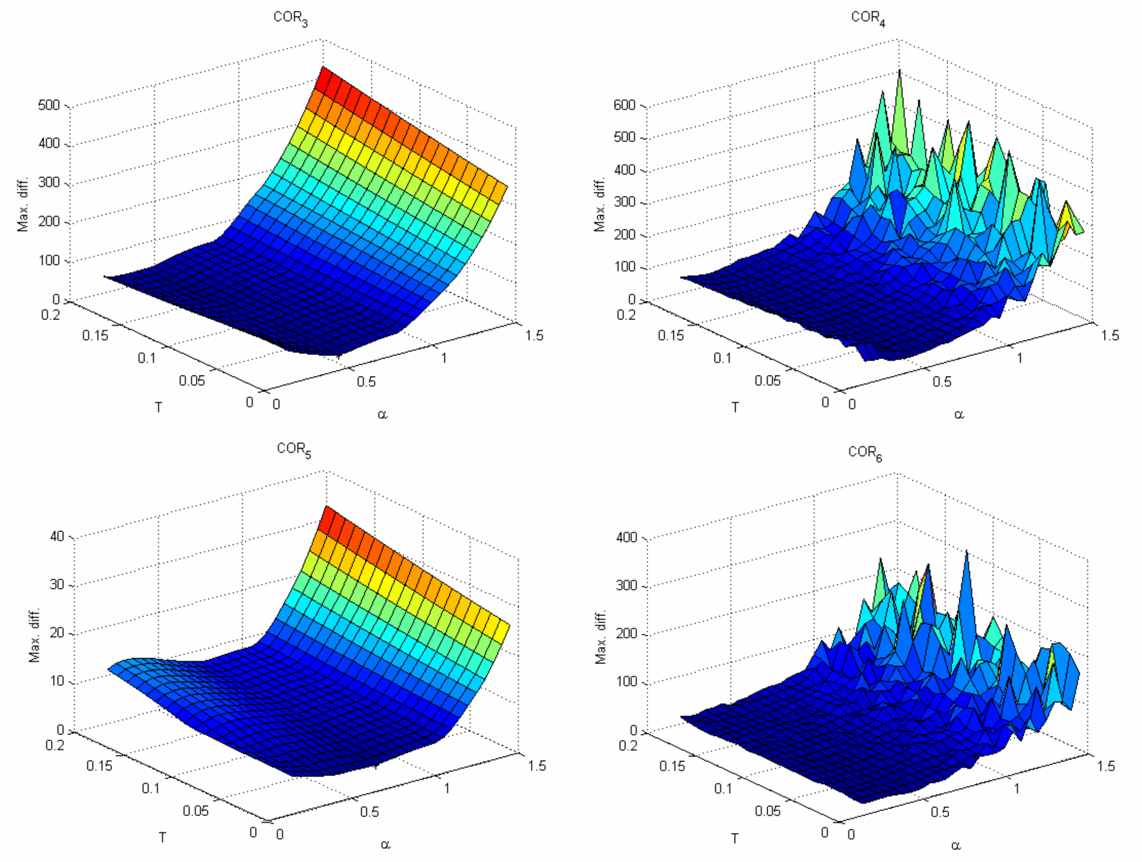

Figure 10. Maximum oscillation = (max(*INV-BL*)-min(*INV-BL*)) surfaces. Q=14 and σ=1.

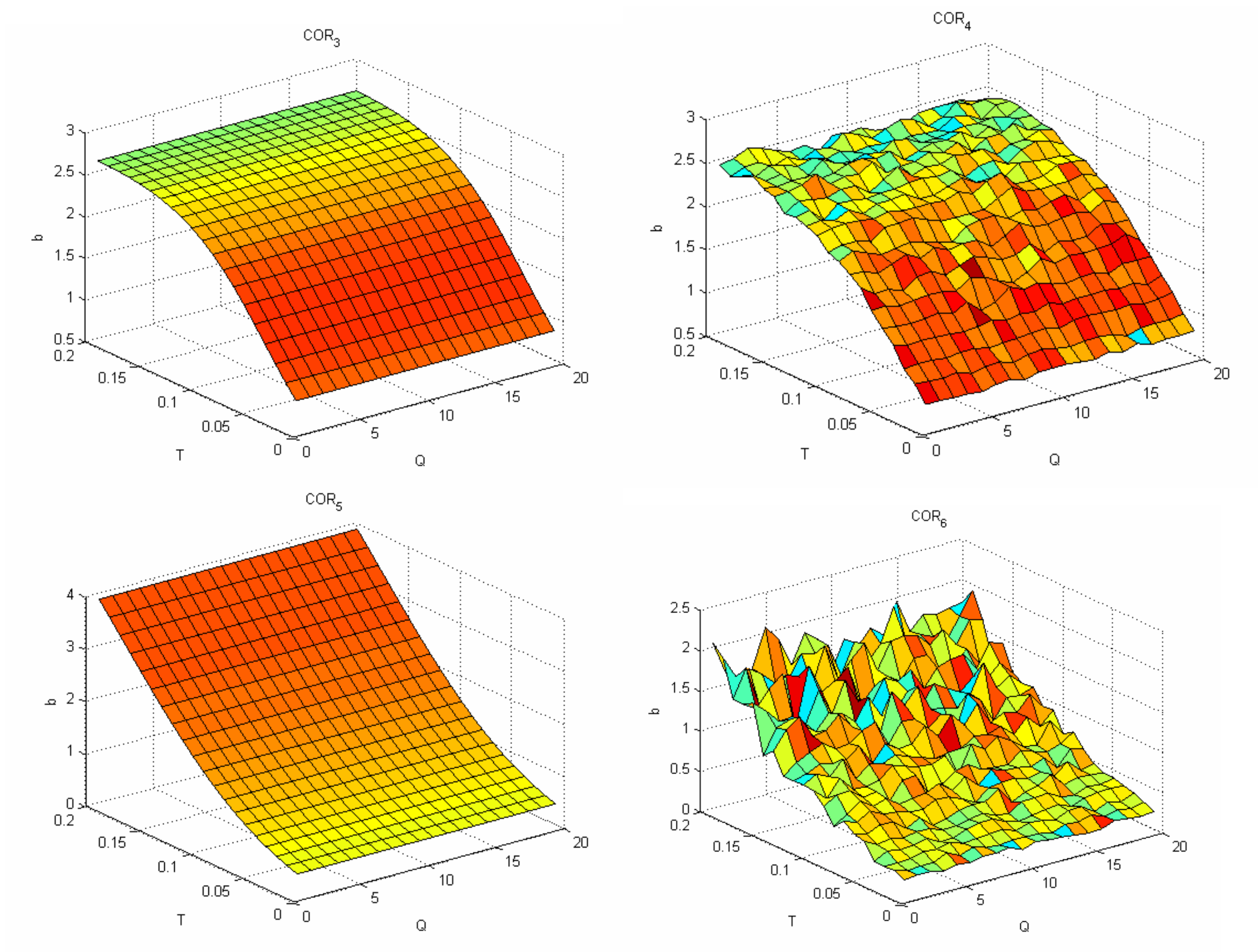

Figure 11. Bullwhip surfaces. Q=14 and σ=1.

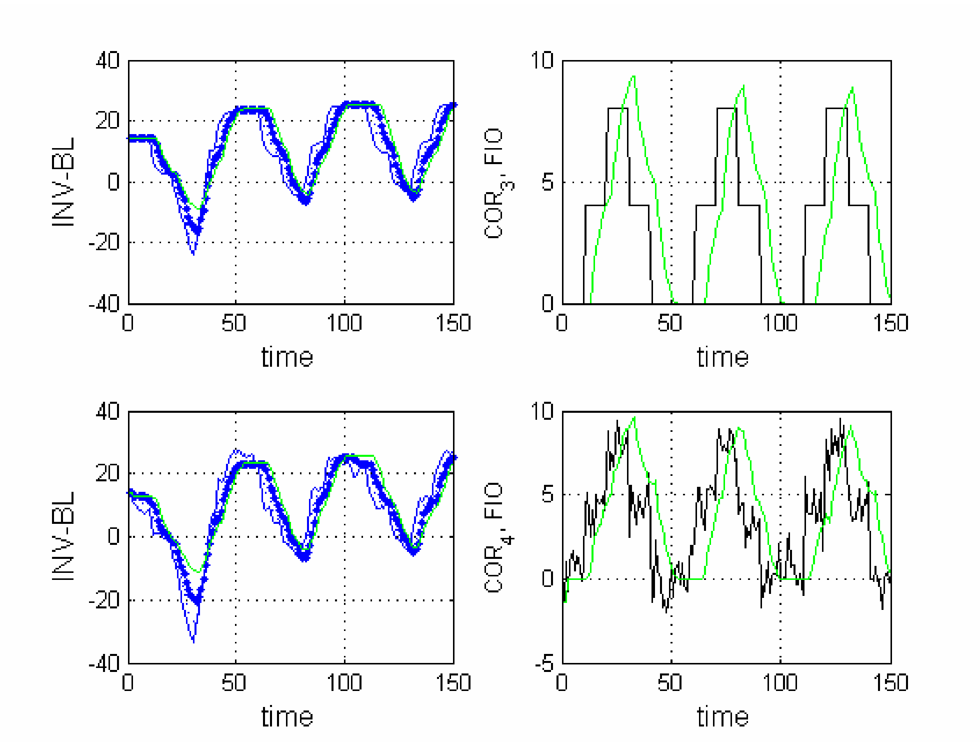

Figure 12. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a step function without (*COR_3*) and with (*COR_4*) noise. $\alpha = 0.3$, $T = 0.03$, $Q = 14$ and $\sigma = 1$.

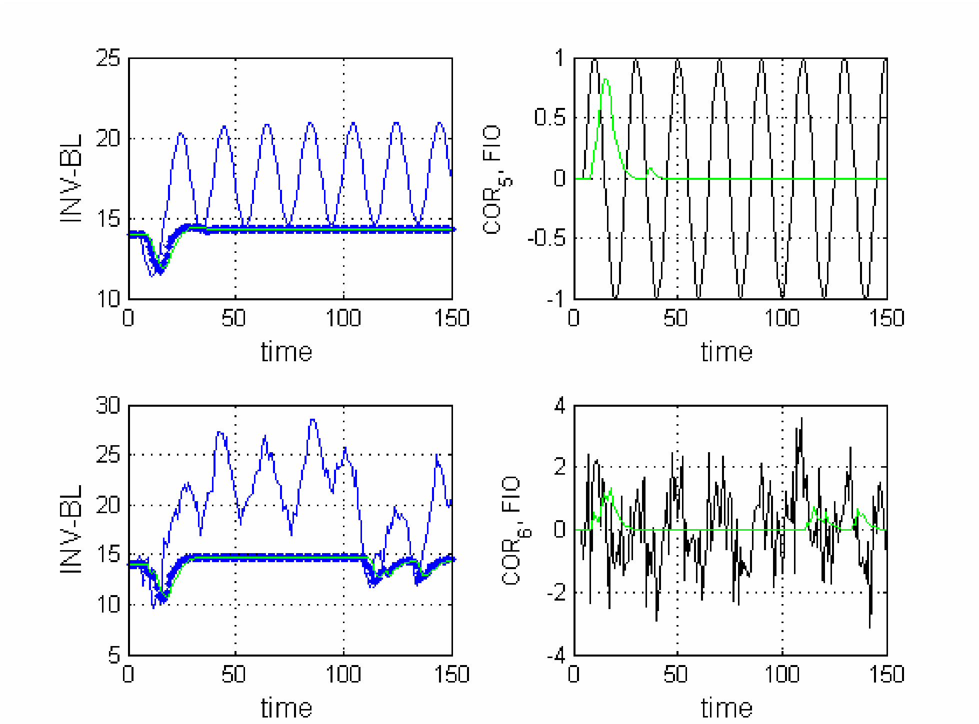

Figure 13. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a step function without (COR_5) and with (COR_6) noise. α = 0.3, T = 0.03, Q = 14 and σ = 1.

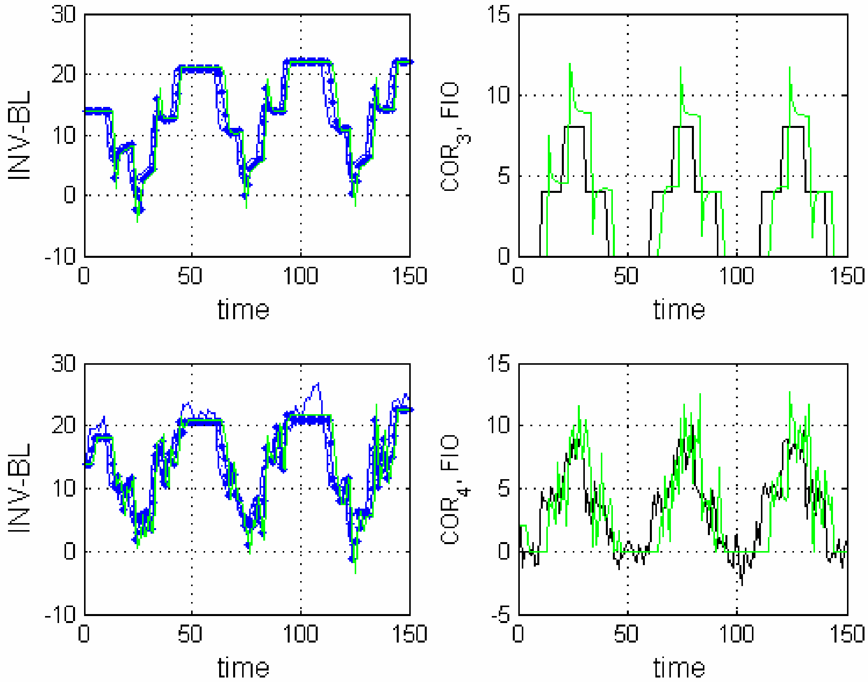

Figure 14. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a step function without (COR_3) and with (COR_4) noise. $\alpha = 0.6$, T=0.03, Q=14 and σ=1.

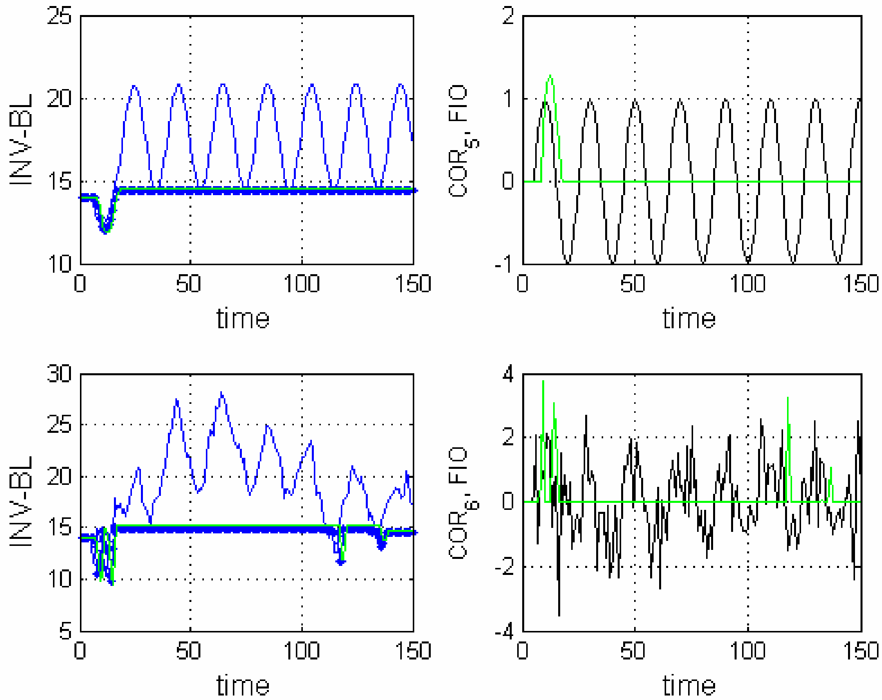

Figure 15. Effective inventories and orders applying the order policy defined by Eqs. (10)-(12) when the customer demand is a step function without (COR_5) and with (COR_6) noise. $\alpha = 0.6$, T=0.03, Q=14 and σ=1.

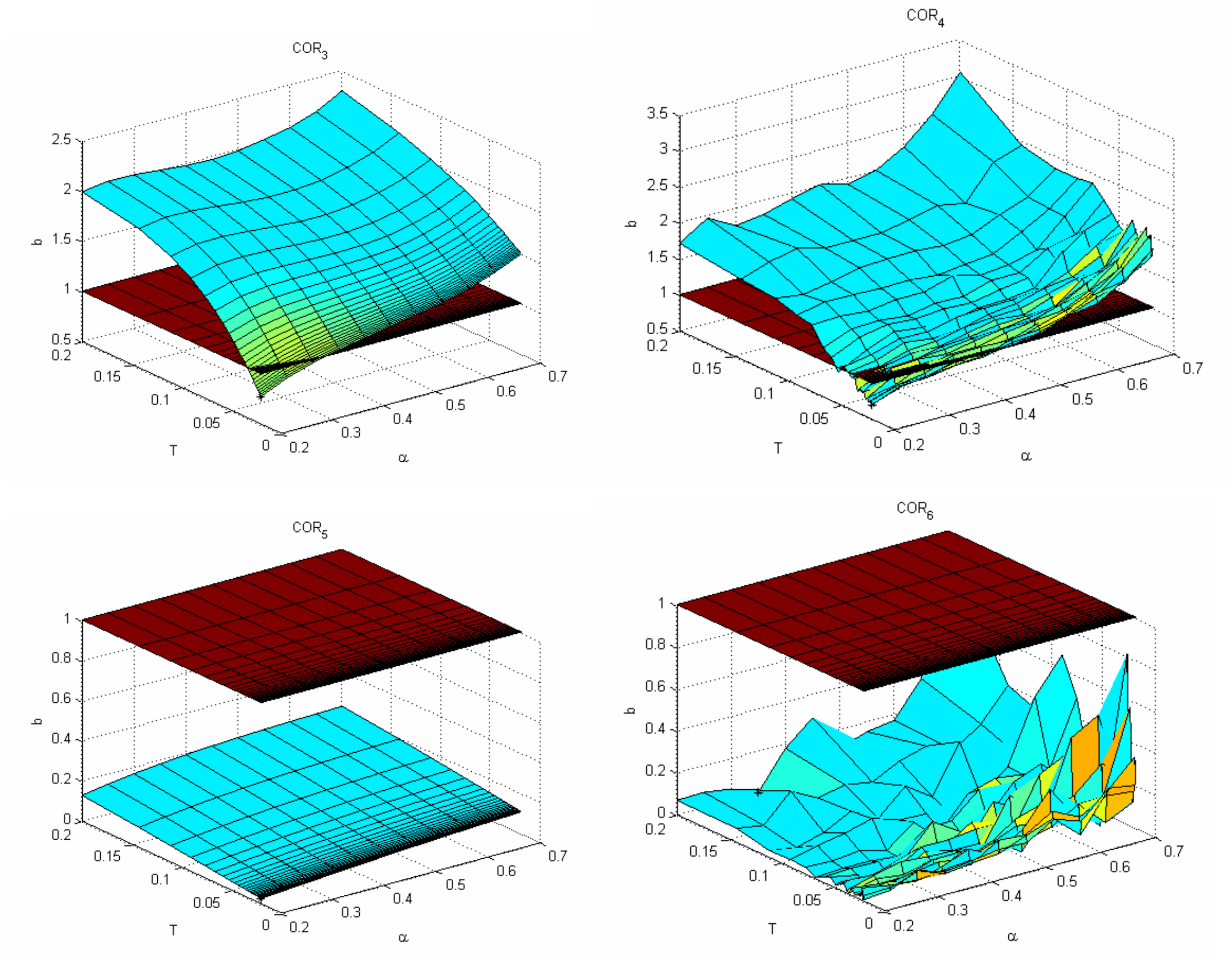

Figure 16. Bullwhip surfaces with COR_3, COR_4, COR_5 and COR_6 translated of four units (COR-4) .
Q=14 and σ=1.

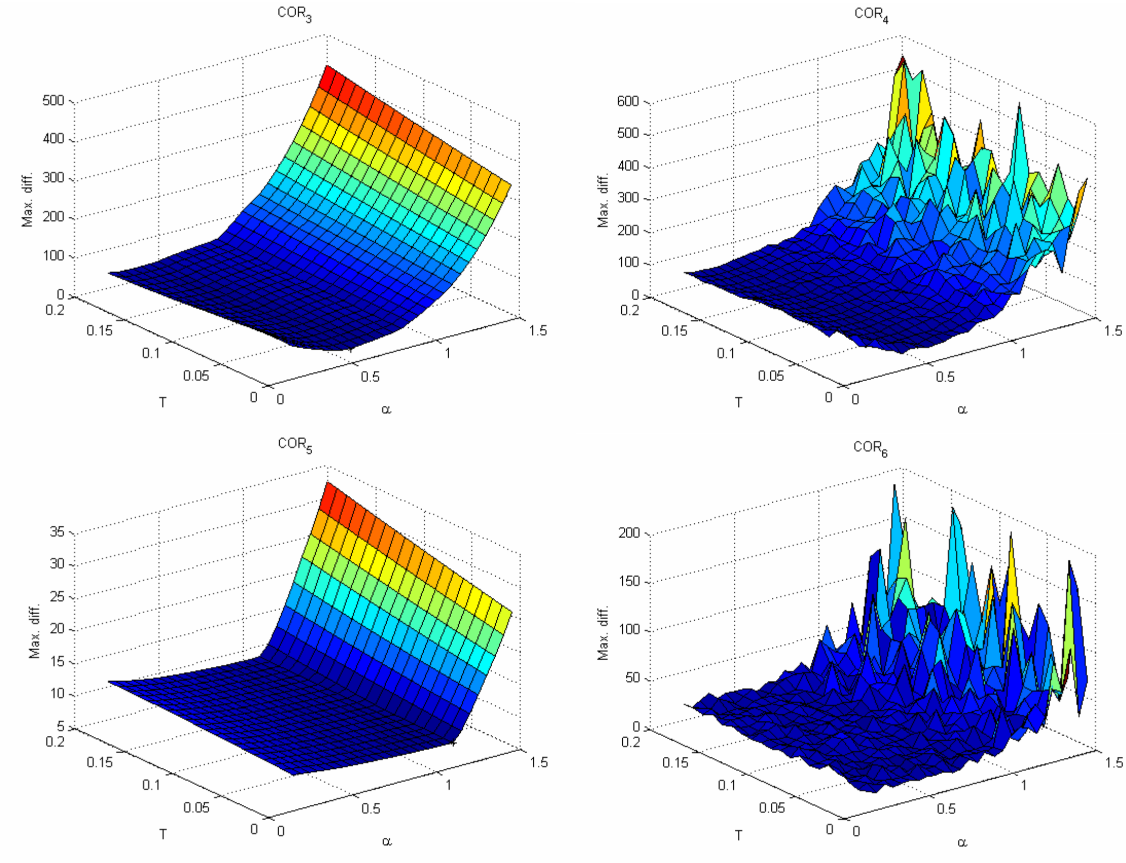

Figure 17. Maximum oscillation = (max(*INV-BL*)-min(*INV-BL*)) surfaces translated of four units (*COR*-4). Q=14 and σ=1.

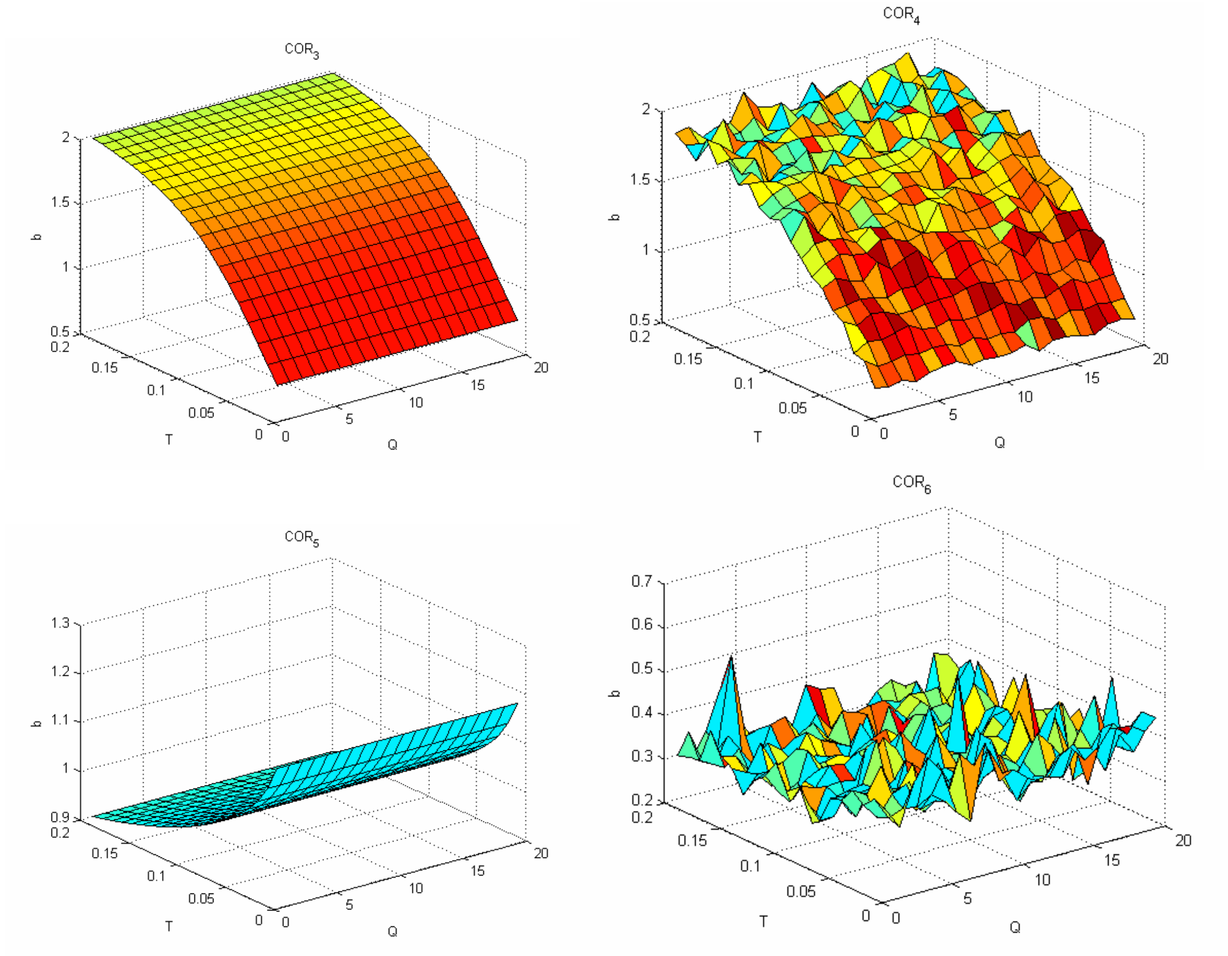

Figure 18. Bullwhip surfaces translated of four units (*COR*-4) . Q=14 and σ=1.

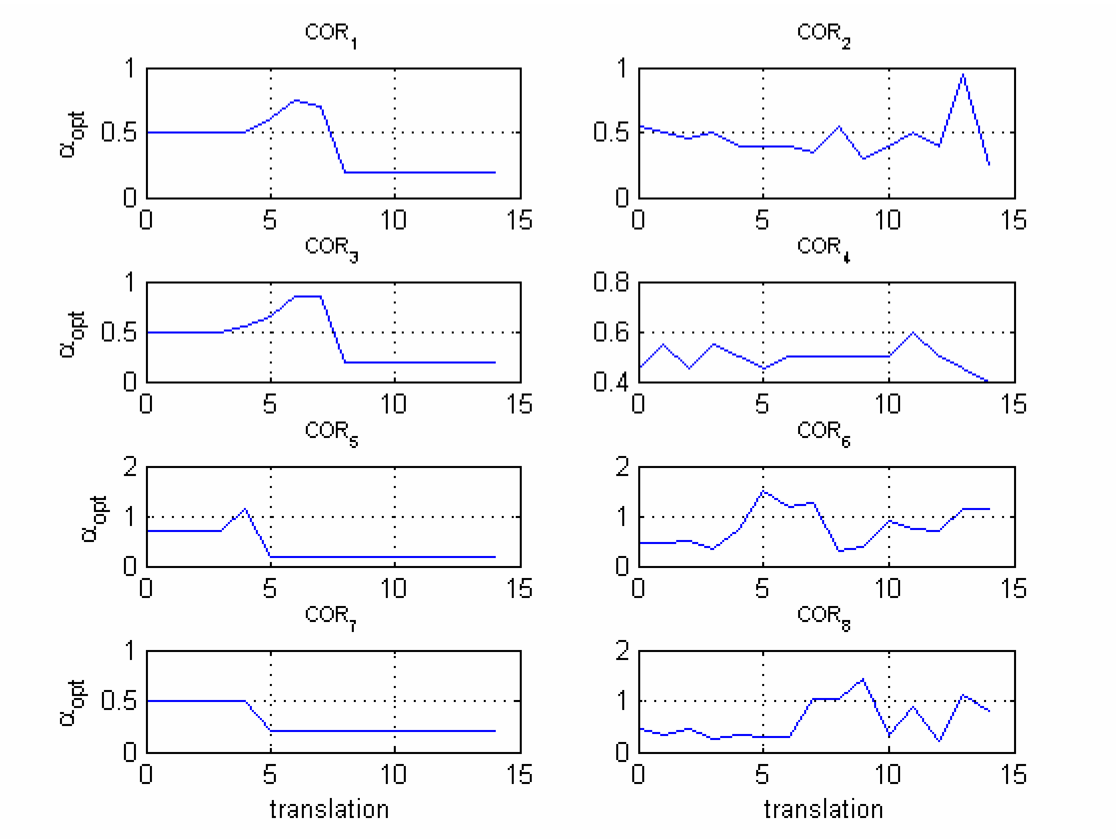

Figure 19. Optimum weight to the history of the demand, α_{opt}, to minimize the maximum difference in the inventories.

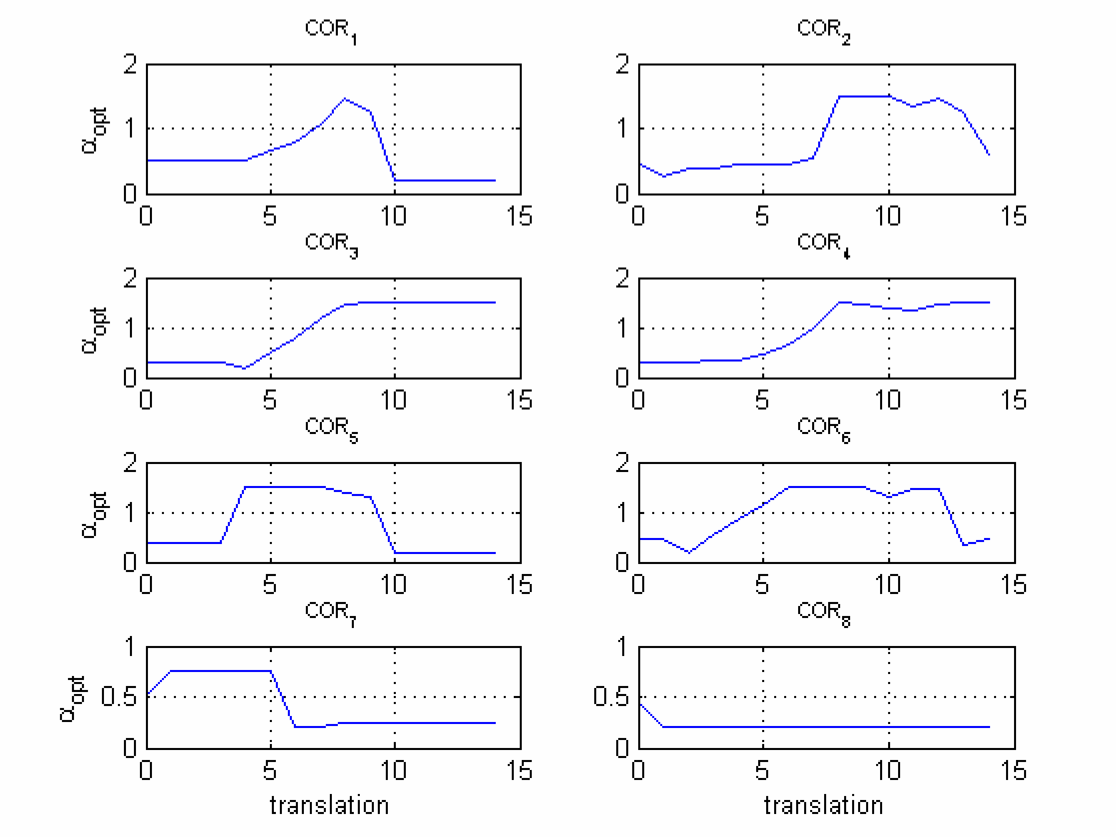

Figure 20. Optimum weight to the history of the demand, α_{opt}, to maintain the bullwhip close to one.

Table 1.Analysed customer demands.

COR_1	Step
COR_2	Step + noise
COR_3	Stairs
COR_4	Stairs + noise
COR_5	Sine
COR_6	Sine + noise
COR_7	Correlated demand
COR_8	Correlated + noise

Table 2. Optimal α parameters, i.e. minima of maxima oscillations surfaces. as a function of the customer demand.

Customer demand type	α
COR_1	0.5
COR_2	0.5
COR_3	0.5
COR_4	0.5
COR_5	0.45
COR_6	0.5
COR_7	0.5
COR_8	0.45