WP6 Vulnerability of Interconnected Networks
Review and Planned activities

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Review:

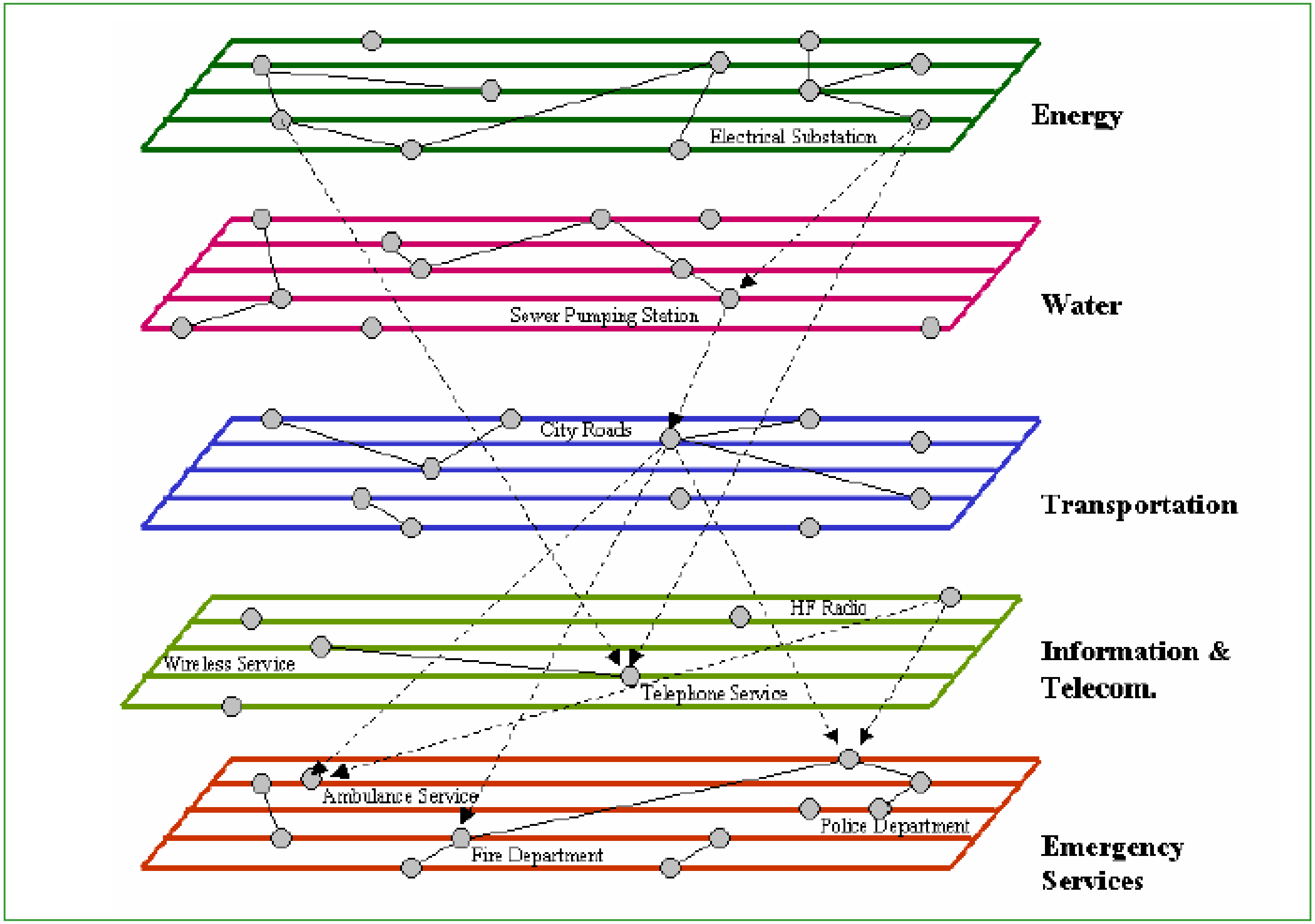
- Models of interconnected networks (infrastructures interdependency)
  - What is infrastructure?
  - Dependency matrix
- What is vulnerability?
- Influence model
  - Binary influence model
  - General influence model
- Planned activities

What is Infrastructure? A network of independent, mostly privately-owned, man-made systems and processes that function collaboratively and synergistically to produce and distribute a continuous flow of essential goods and services.

[From: President's Commission on Critical Infrastructure Protection, Critical Foundations: Protecting America's Infrastructures (1997). Available: **www.ciao.gov**]

Eight infrastructures: telecommunications, electric power systems, natural gas and oil, banking and finance, transportation, water supply systems, government services, and emergency services.

Interdependency: A bidirectional relationship between two infrastructures through which the state of each infrastructure influences or is correlated to the state of the other.



Infrastructure interdependencies

Infrastructure Interdependency Modeling:

Be it through direct connectivity, policies and procedures, or geospatial proximity, most *infrastructure systems* interact. These interactions often create complex relationships, dependencies, and interdependencies that cross infrastructure boundaries.

The modeling and analysis of interdependencies between infrastructures is a relatively new and very important field of study.

From: *Critical Infrastructure Interdependency Modeling: A Survey of U.S. and International Research*, P. Pederson, D. Dudenhoeffer, S. Hartley, M. Permann,

August 2006, Idaho National Laboratory, Critical Infrastructure Protection Division

Sector	Element	Energy & Utilities					Services		
		Electrical Power	Water Purification	Sewage Treatment	Natural Gas	Oil Industry	Customs and Immigration	Hospital & Health Care Services	Food Industry
Energy & Utilities	Electrical Power		L			M			
	Water Purification	H				M			
	Sewage Treatment	M	H			H			
	Natural Gas	L				L			
	Oil Industry	H	L						
Services	Customs & Immigration	H	L	L	L	L		L	
	Hospital & Health Care Services	H	H	L	H	H	M		H
	Food Industry	H	H	H	L	M	M	L	

Key: H High, M Medium, L Low

The Critical Infrastructure Protection Task Force of Canada used a dependency matrix to relate the interdependency among six sectors identified as crucial: Government, Energy and Utilities, Services, Transportation, Safety, and Communications.

Dependency matrix

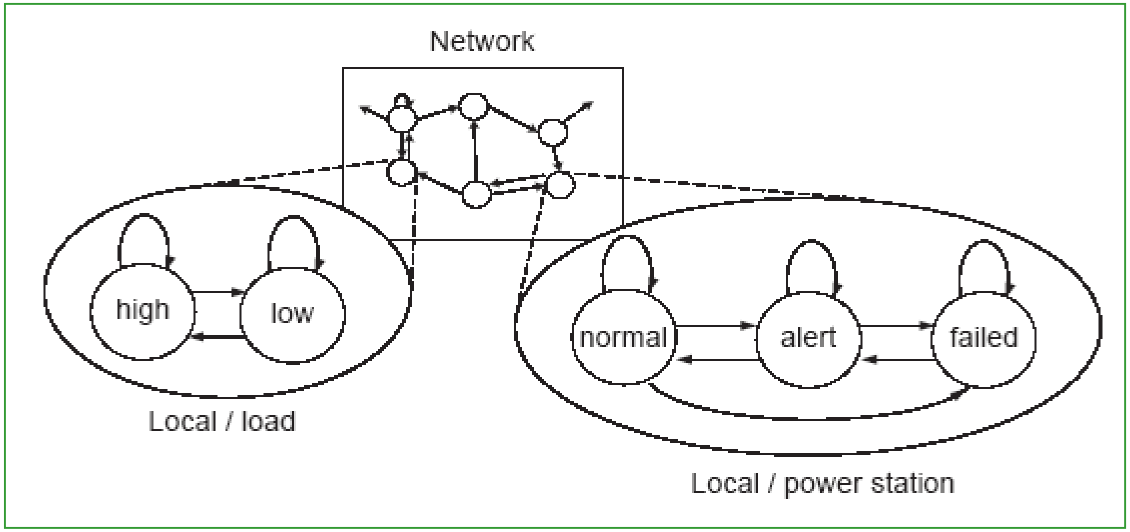
(Do not touch the grey fields. You are welcome to a

	Dependency matrix			Effects	
	Electricity	Data communication	Payment systems	EFFECT IN	EFFECT OUT
Electricity	89,0	10,0	1,0	333,3	630,0
Data communication	80,0	19,0	1,0	333,3	230,0
Payment systems	20,0	40,0	40,0	333,3	140,0

National Emergency Supply Agency (NESA), Hannu Sivonen

Influence model

Influence model is a stochastic dynamical system defined on a graph, and is described at two levels: the network level and the local level.



At the network level, each node can be treated as one active entity, and is called a site.

Example: A site can either be a power station (generator) or a load. A power station may be represented as being in one of three possible statuses at any given time: normal, alert or failed. The loads, which can be cities or factories, might be in either high or low status, depending on the present level of demand.

Directed graphs

Let $A = [a_{ij}]$ be an $n \times n$ matrix

Directed graph $\Gamma(A)$:

Directed edge from i to j exsits if and only if $a_{ij} \neq 0$

Edge weight is given by a_{ij}

Stochastic matrix, Transpose matrix, Directed (weighted) graph = Network

Binary influence models

Binary influence model: the status of each node at any given time step is assumed to be 0 or 1, which may represent any two different statuses such as ‘on’ vs. ‘off’, ‘healthy’ vs. ‘sick’, or ‘normal’ vs. ‘failed’.

The binary influence model can potentially illuminate our understanding of the qualitative behavior of a number of systems.

In power systems, this model can be used as a highly simplified paradigm for cascading blackouts. Here the network graph would represent the power grid, and each node would be a substation or a power plant whose status value is amenable to a binary label. To simulate cascading failure, we can start with a network in which every node is in ‘normal’ state and then initiate a node failure by turning the status at some node to ‘failed’.

Binary Influence Model

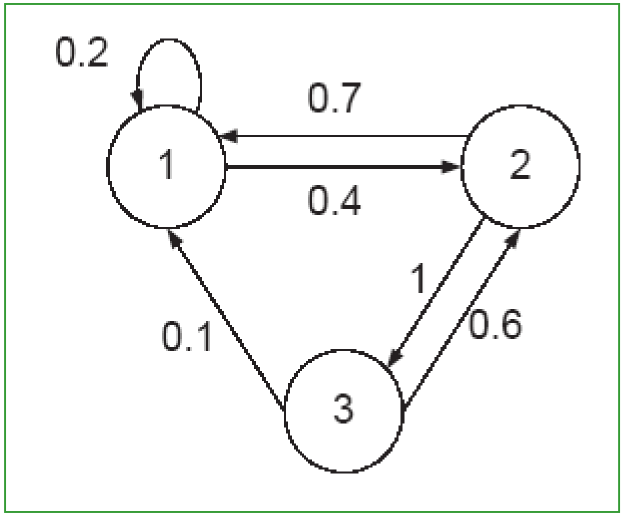
D $n \times n$ stochastic matrix

$\Gamma(D^T)$ network influence model

The sum of edges pointing into a site is 1. This feature allows us to treat each edge weight as the ***relative amount of influence*** from the source node to the destination.

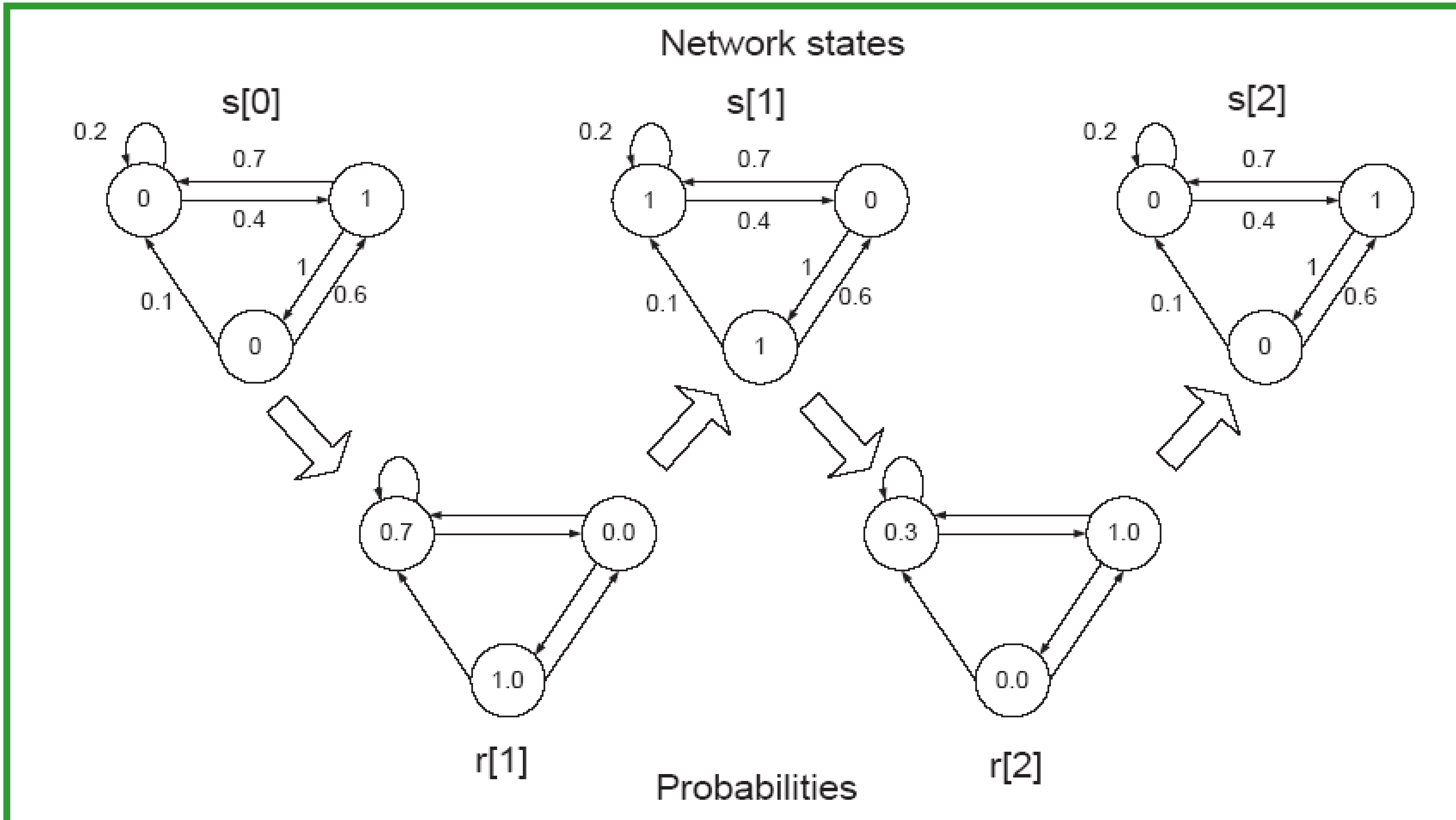
$$D' = \begin{bmatrix} .2 & .4 & 0 \\ .7 & 0 & 1 \\ .1 & .6 & 0 \end{bmatrix}$$

Transpose matrix



$$\mathbf{s}[k] \stackrel{\triangle}{=} [\, s_1[k] \;\ldots\; s_n[k]\,]'$$

$$\begin{aligned} \mathbf{r}[k+1] &= D\,\mathbf{s}[k] \\ \mathbf{s}[k+1] &= Bernoulli(\mathbf{r}[k+1]) \end{aligned}$$



If D is ergodic, then $$\lim_{k \to \infty} D^k = \mathbf{1}\pi^T$$

π is the left eigenvector corresponding to the eigenvalue at 1, which has been normalized so that

$$\pi^T \mathbf{1} = 1$$

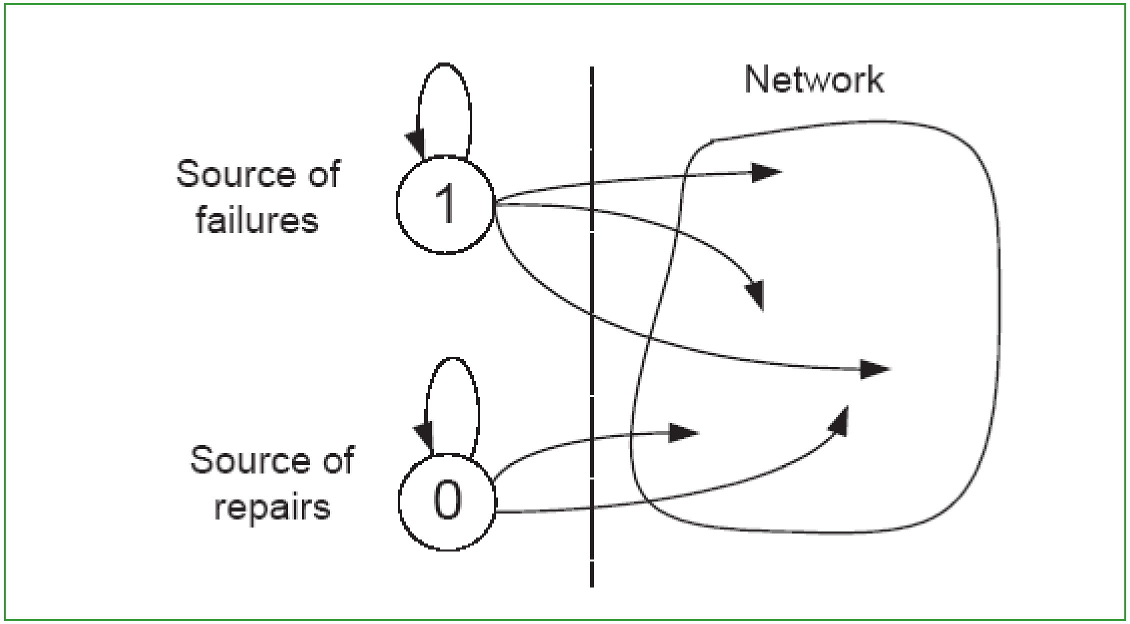
$\Gamma(D^T)$ is ergodic

The probability that the influence process starting from initial state $s[0]$ will eventually settle in the all-ones consensus state is $\pi^T s[0]$

The probability of reaching the all-zeros consensus state is $1 - \pi^T s[0]$

Open problem: how does the basin of attraction of the consensus state depend on the network topology?

Evil-Rain model



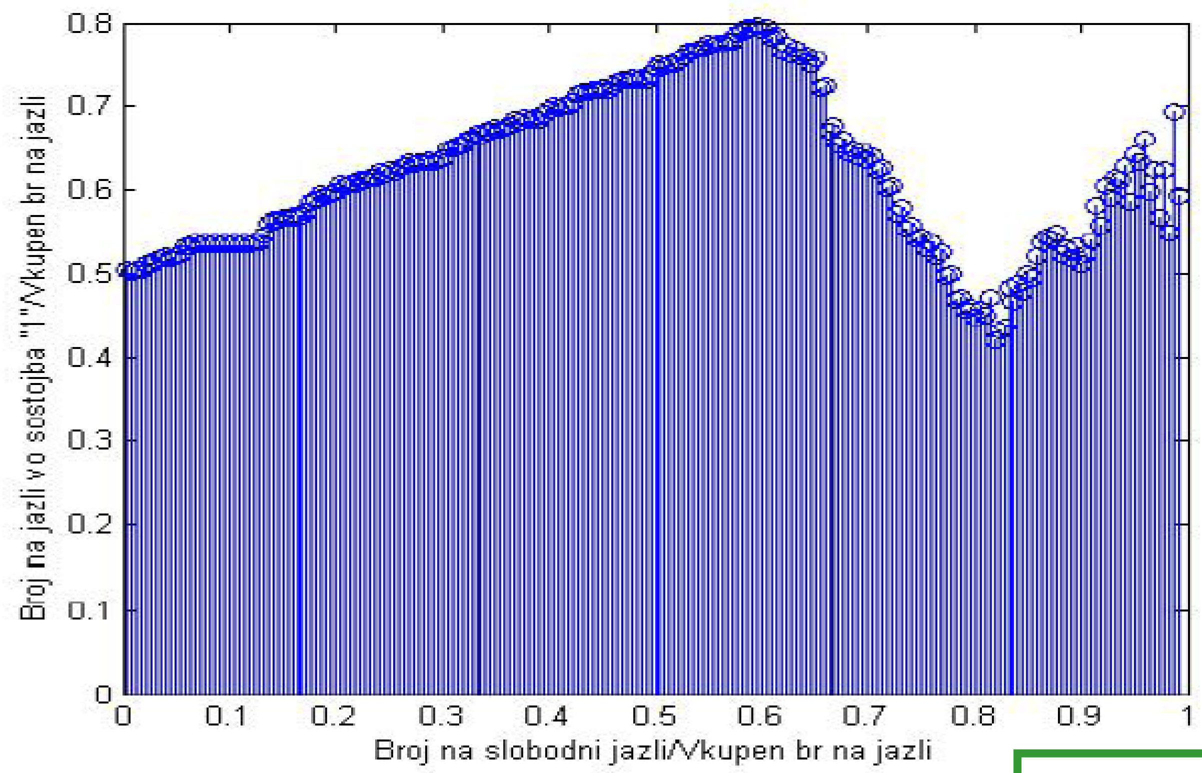
$$D = \begin{bmatrix} 1 & & \\ & 1 & \\ \mathrm{e}_1 & \mathrm{e}_2 & F \end{bmatrix}, \qquad \mathrm{s}[k] = \begin{bmatrix} 1 \\ 0 \\ \widetilde{\mathrm{s}}[k] \end{bmatrix}$$

The expected number of status-1 sites in steady-state is

$$\mathbf{1}'(I - F)^{-1}\mathbf{e}_1 .$$

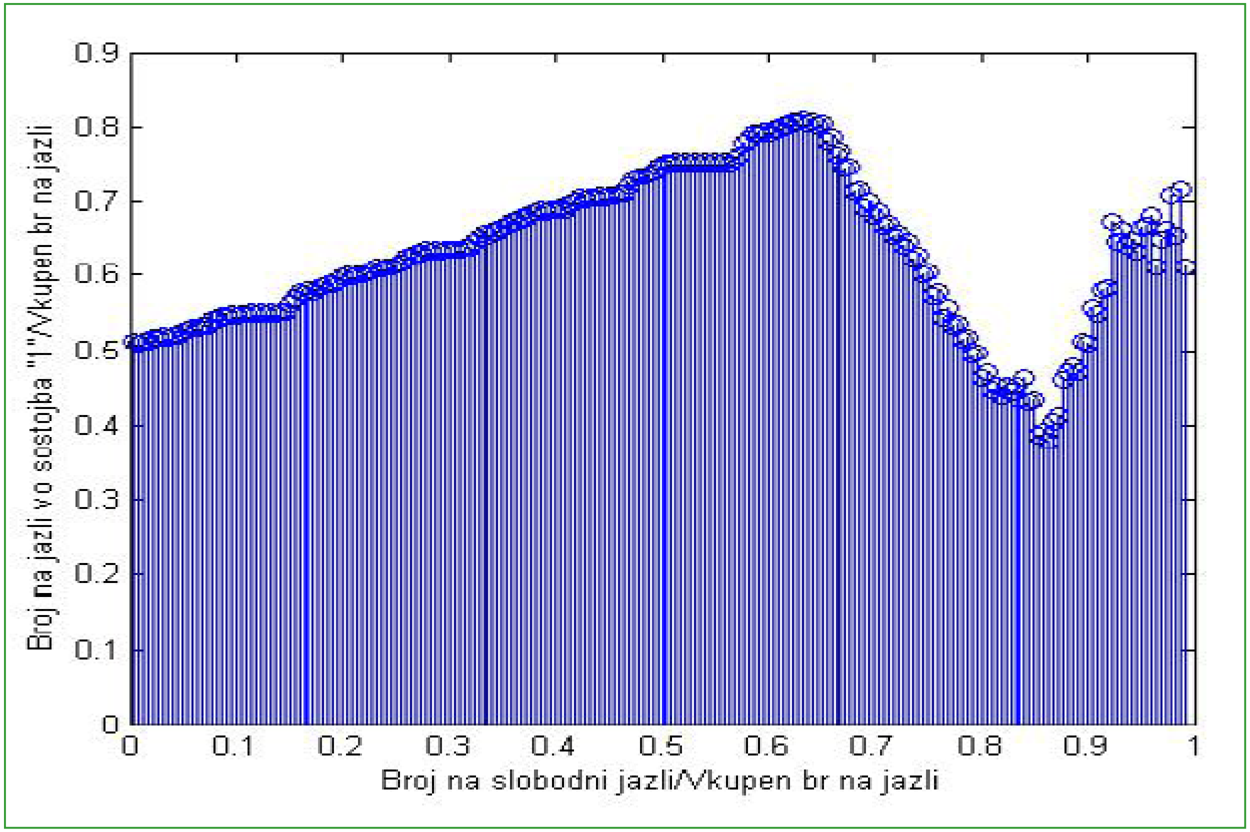
Open problem: how does the behavior of the evil-rain model depend on the network topology?

y = Number of nodes in the state 1/ Total number of nodes

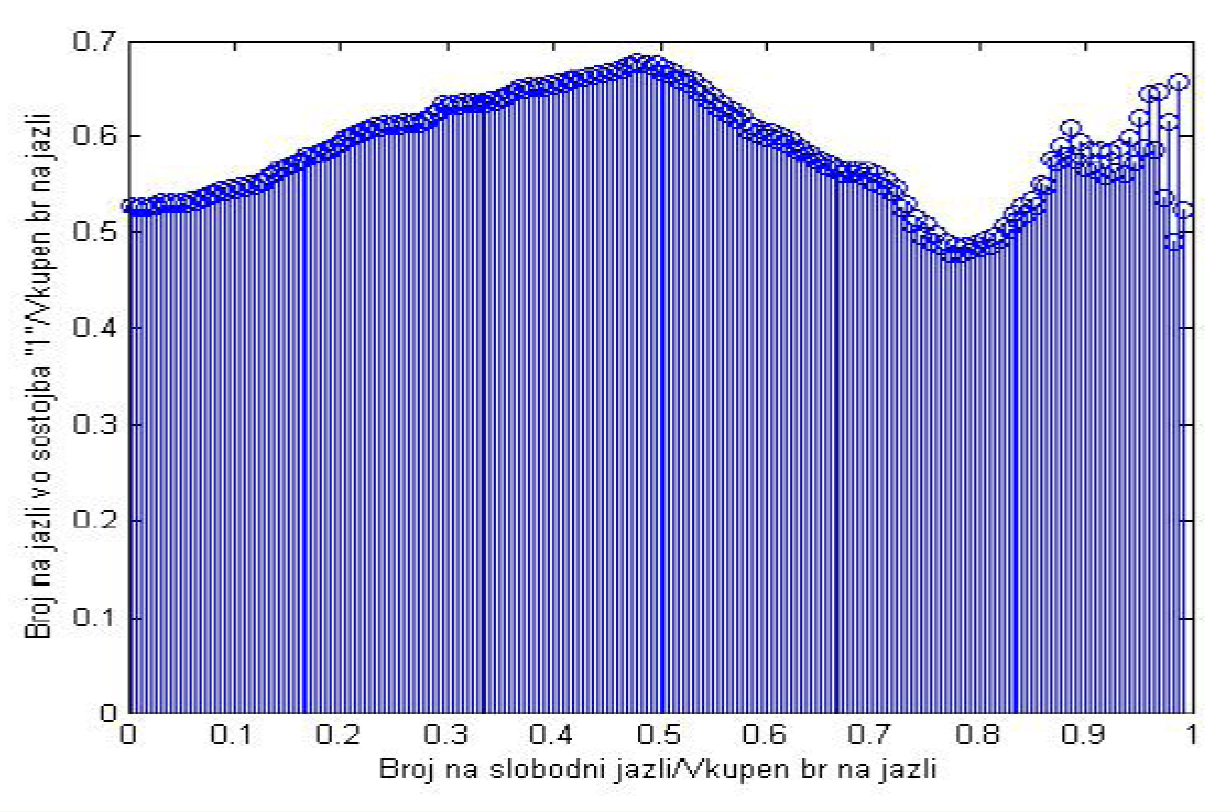


SMALL WORLD $v_1 = v_0 = 0.9$

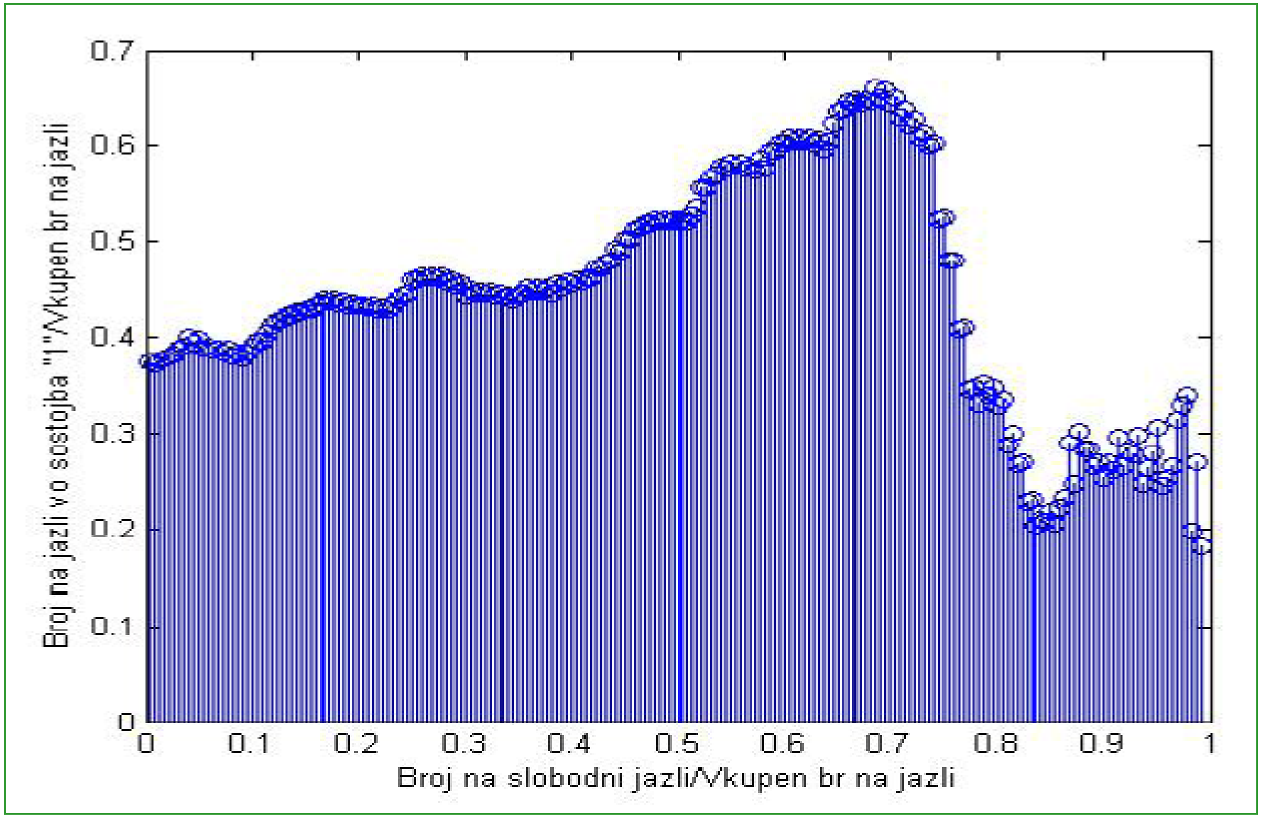
x = Number of uninfluenced nodes/Total number of nodes



SMALL WORLD $v_1 = v_0 = 0.5$



SMALL WORLD $v_1 = v_0 = 0.1$



SMALL WORLD $v_1 = 0.1 v_0 = 0.9$

Generalization

D is stochastic matrix

$$\mathbf{r}[k+1] = D\,\mathbf{s}[k]$$
$$\mathbf{s}[k+1] = Bernoulli(\mathbf{r}[k+1])$$

$$x(k+1) = Bernoulli[ADx(k)+(I-A)s]$$
$$y(k+1) = ADy(k)+(I-A)s$$

$$A = diag(a_1,...,a_n) \quad s = [s_1,...,s_n]^T$$

d_{ii} measures the amount of influence that *i* exerts on *i* (relative to the total amount of influence that *i* receives).

a_i describes the '**strength of influence**' of the cite *i*.

s_i plays role of the '**confidence**' of the site *i* to the values 0 and 1.

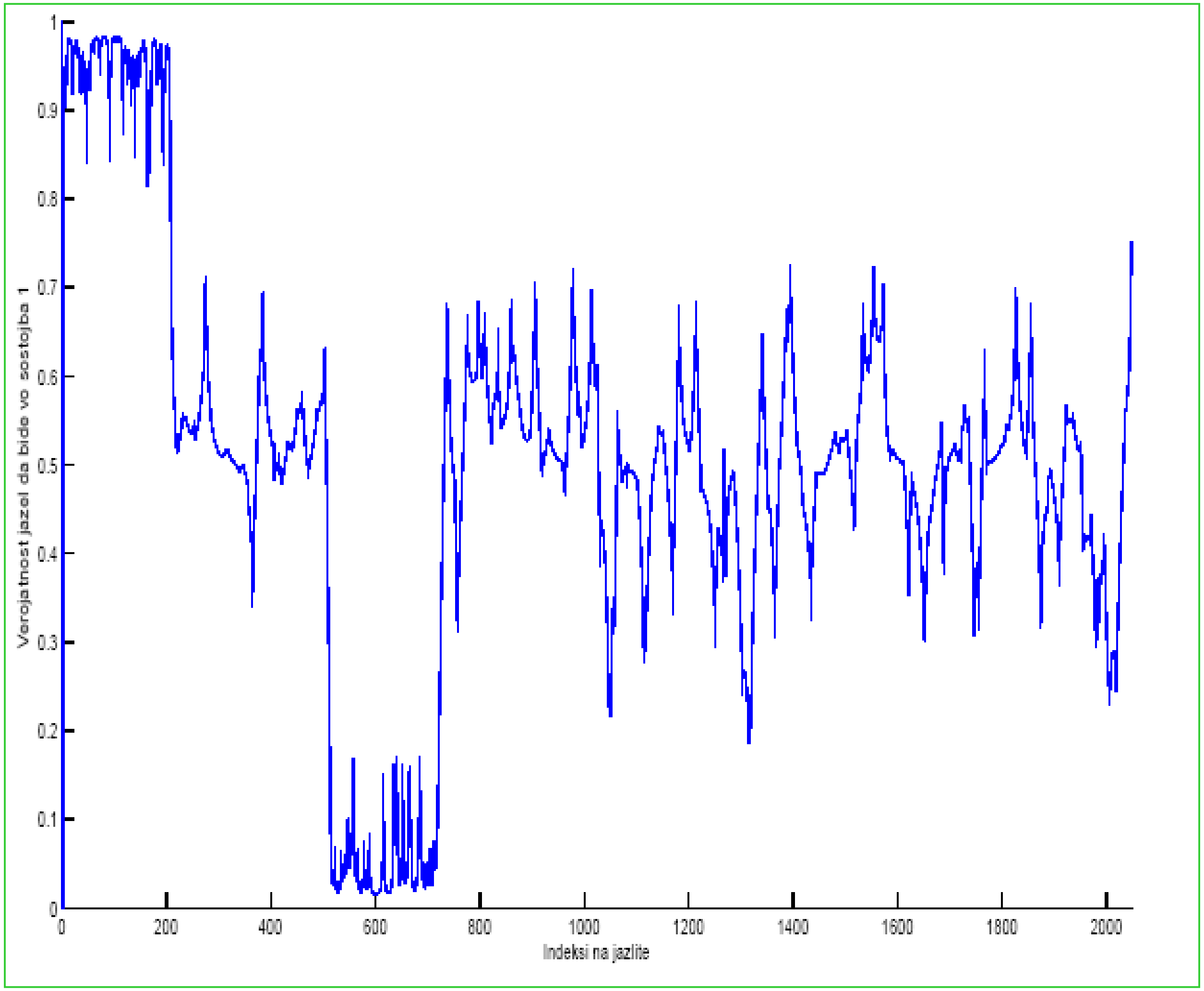
The parameter a_i describes the **strength of influence** of the cite i.

$$y(k+1) = ADy(k) + (I - A)s$$

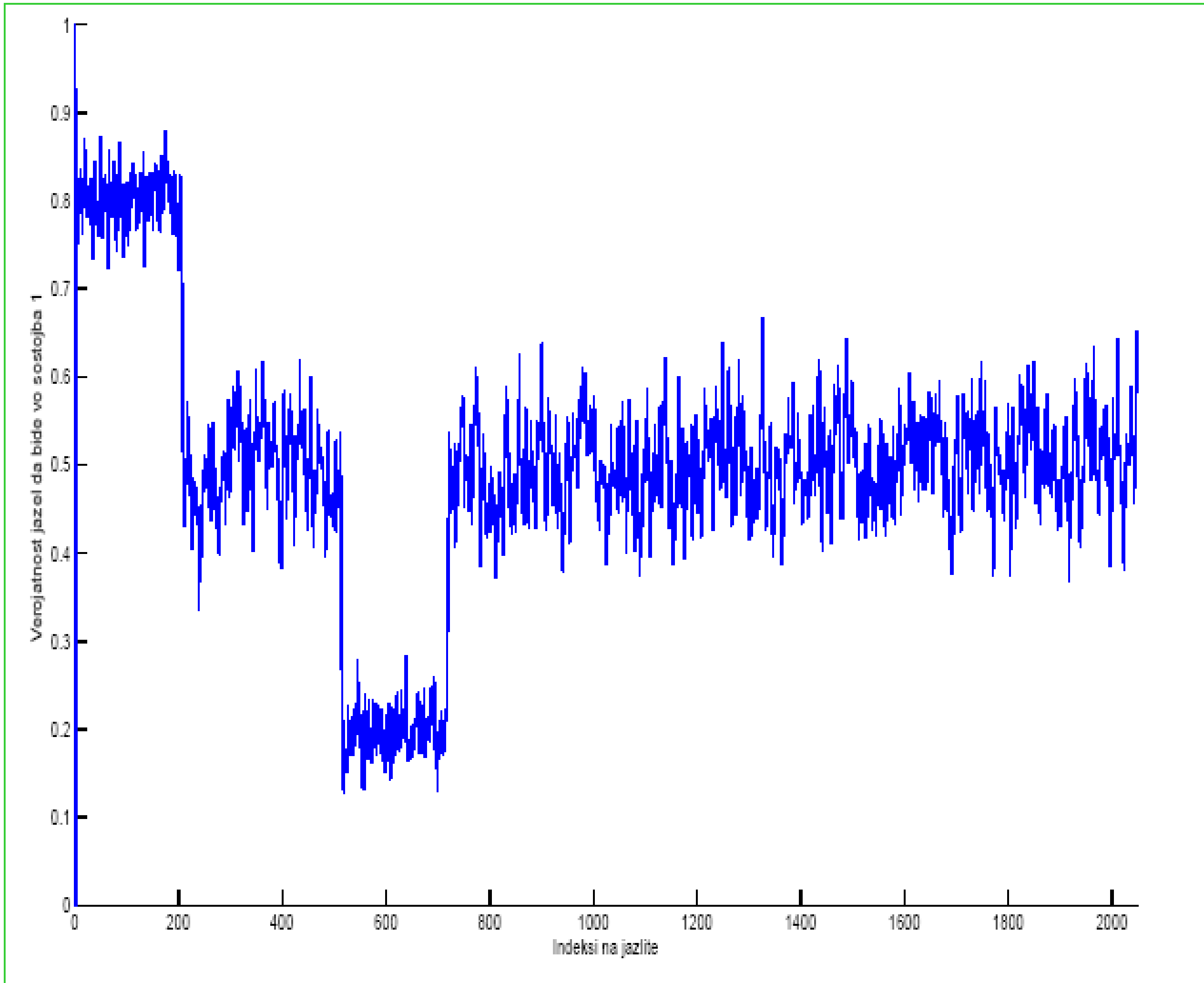
If a_i is close to 1, the status of the site *i* at the time *k*+1, $x_i(k+1)$, depends mostly on the total amount of the influence that the cite *i* receives at time *k* and not on the parameter s_i.

When a_i is close to 0, the influence of other sites to the status of the site *i* is very small: the status of the cite *i* is mostly influenced by the value of s_i (which is constant), and therefore, the status of the cite *i* does not change in time rapidly.

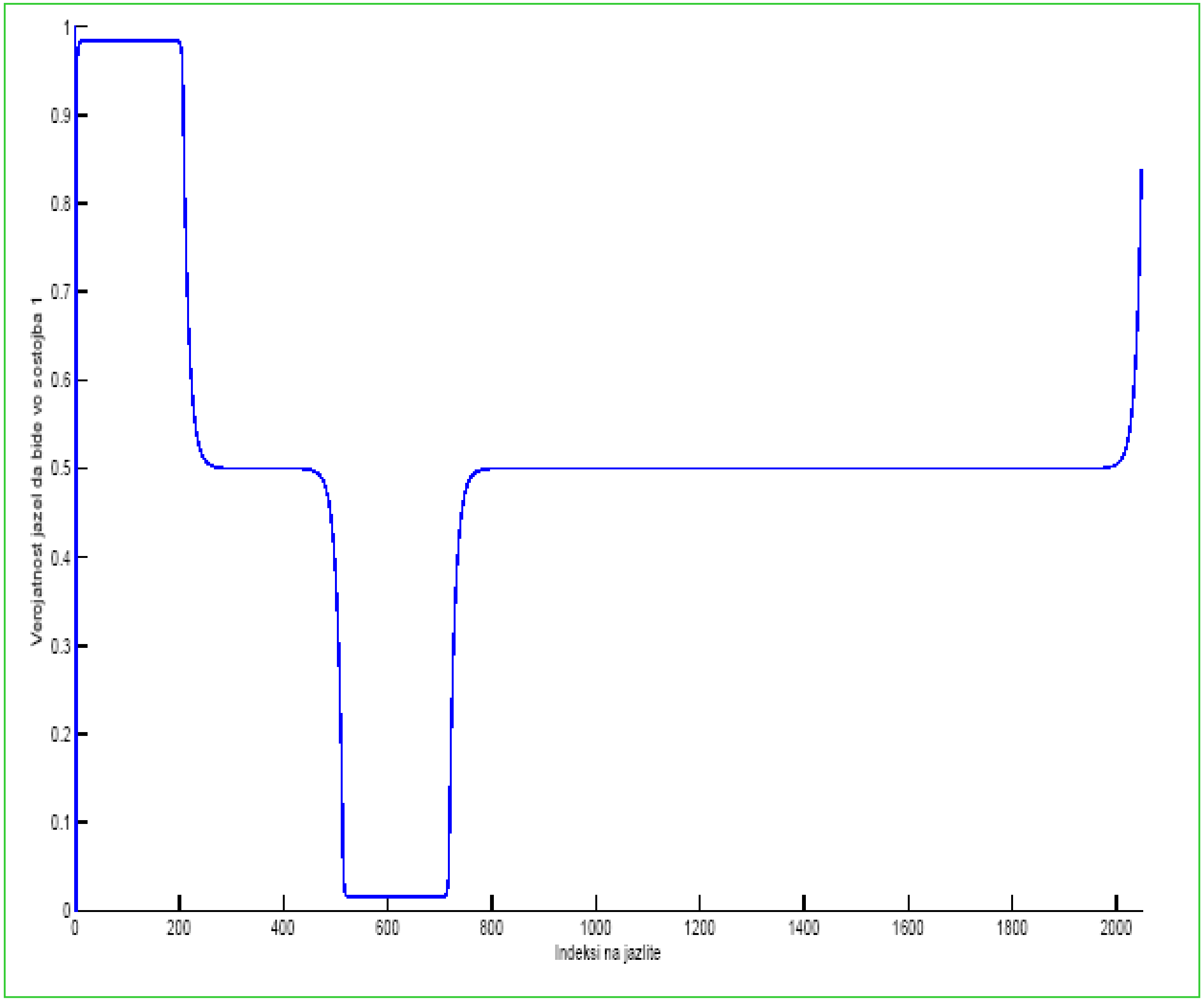
The parameter s_i plays role of the **confidence** of the site *i* to the values 0 and 1: if $|1-2s_i|$ is close to 1, the state i is '<u>confident</u>' to the values 0 or 1, while when $|1-2s_i|$ is close to 0, the site i is '<u>uncertain</u>' about the values 0 and 1.



small world $a = b = 10$; $v_1 = v_0 = 0.3$



random graph $a = b = 10$; $v_1 = v_0 = 0.3$



regular graph $a = b = 10$; $v_1 = v_0 = 0.3$

$$y(k+1) = ADy(k) + (I - A)s$$

$$\lim_{k\to\infty} y(k) = \begin{cases} (I-G)^{-1}Bs & 0 < a_j < 1, \forall j \\ D^{\infty} y(0) & a_j = 1, \forall j \\ s & a_j = 0, \forall j \\ G^{\infty} y(0) & \text{otherwise} \end{cases}$$

$$B = I - A, G = AD$$

$$A = diag(a_1, \ldots, a_n) \quad s = [s_1, \ldots, s_n]^T$$

D is nonnegative matrix

$$d_{ij} \geq 0$$

$$x_i(k+1) = \begin{cases} 1 & \sum_{j=1}^{n} d_{ij} x_j(k) > 0 \\ -1 & \sum_{j=1}^{n} d_{ij} x_j(k) \leq 0 \end{cases}$$

Similar to Threshold Model, see D. J. Watts, *A simple model of fads and cascading failures*, Santa Fe Institute Working Paper, 2000.

Open problem: how to generalize the model for heterogeneous networks

$$D = \begin{bmatrix} 0.95 & 0.01 & 0 & 0 & 0.04 \\ 0.2 & 0.76 & 0 & 0 & 0.04 \\ 0.04 & 0.2 & 0.56 & 0 & 0.2 \\ 0.01 & 0 & 0 & 0.98 & 0.01 \\ 0.2 & 0.01 & 0 & 0 & 0.79 \end{bmatrix}$$

$$L = 0.01, M = 0.04, H = 0.2.$$

Electrical, Water, Sewage, Gas, Oil

$$p = [0.8000 \quad 0.8000 \quad 0.8000 \quad 0.8000 \quad 0.8000]$$

$$p = [0.0400 \quad 0.0400 \quad 0.0400 \quad 0.0400 \quad 0.0400]$$

$$p = [0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$p = [0 \quad 0 \quad 0 \quad 0 \quad 0]$$

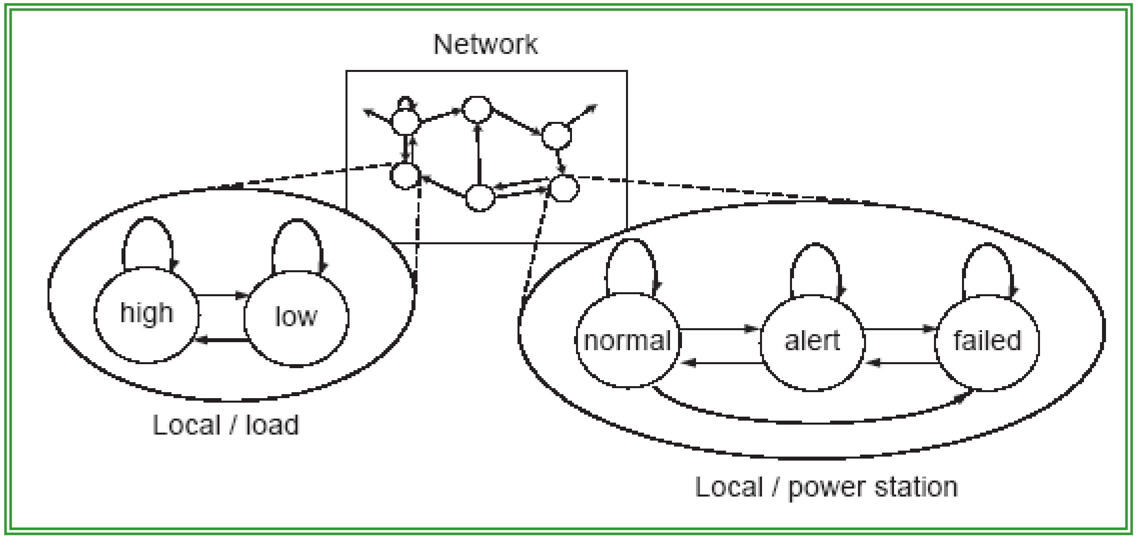
$$p = [0.1600 \quad 0.1600 \quad 0.1600 \quad 0.1600 \quad 0.1600]$$

General influence model

The influence model is a discrete-time Markov process whose state space is the tensor product of the statuses of all the local Markov chains.

$$H \triangleq D' \otimes \{A_{ij}\} = \begin{bmatrix} d_{11}A_{11} & \cdots & d_{n1}A_{1n} \\ \vdots & & \vdots \\ d_{1n}A_{n1} & \cdots & d_{nn}A_{nn} \end{bmatrix}$$

$$\begin{aligned} \mathbf{p}'[k+1] &\triangleq \mathbf{s}'[k]\,H \\ \mathbf{s}'[k+1] &\triangleq MultiRealize(\,\mathbf{p}'[k+1]\,) \end{aligned}$$



High	Low

$$A_{11} = \begin{bmatrix} 1-p & p \\ 1-q & q \end{bmatrix}$$

Normal	Alert	Failed

$$A_{22} = \begin{bmatrix} a & b & 1-a-b \\ c & d & 1-c-d \\ e & f & 1-e-f \end{bmatrix}$$

Load Plant

$$D^T = \begin{bmatrix} 1-t & 1 \\ t & 0 \end{bmatrix}$$

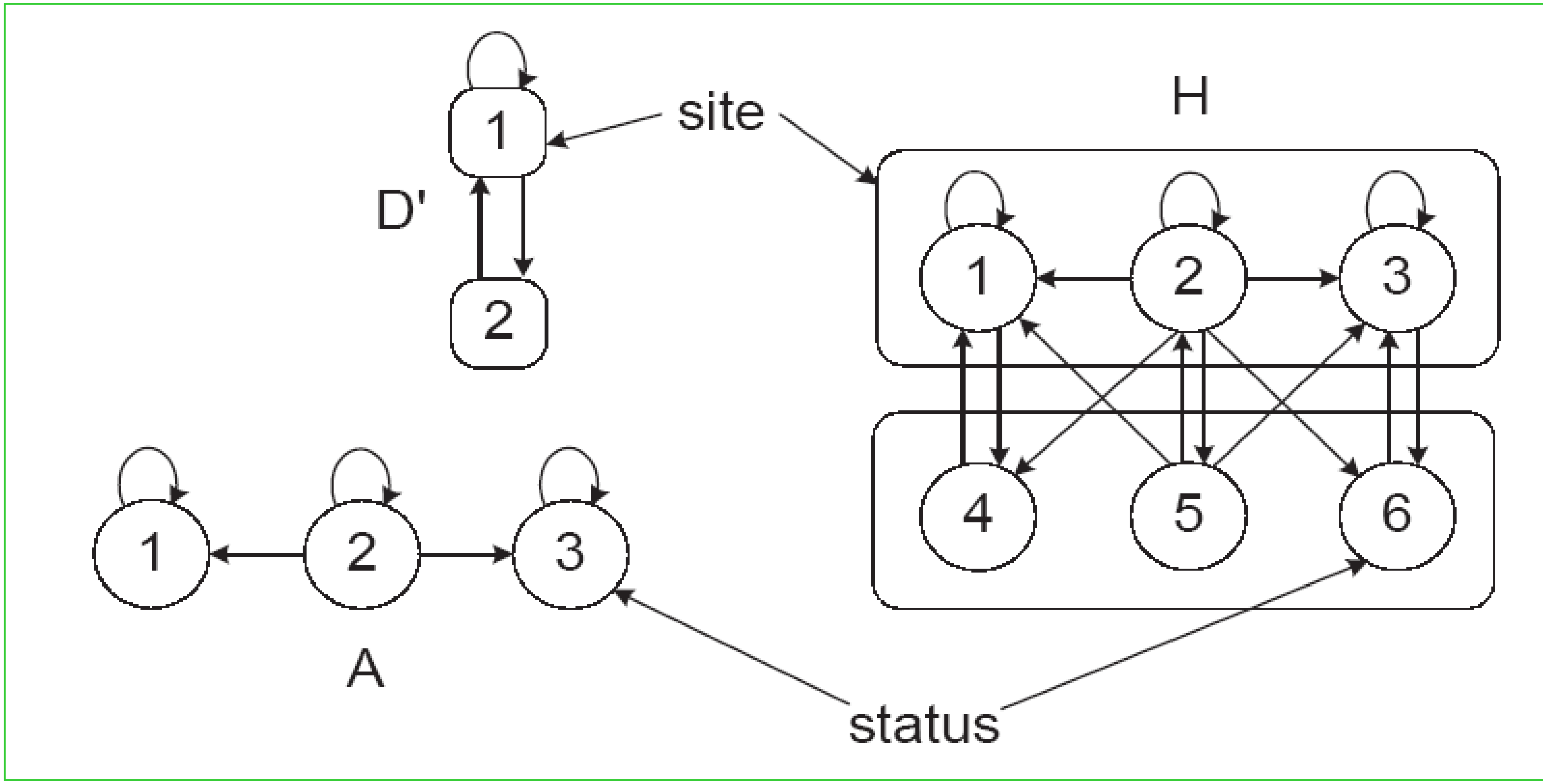
Load

Plant

Influence matrix

$$H = D^T \otimes \{A_{ij}\}$$

$$H = \begin{bmatrix} (1-t)(1-p) & (1-t)p & 0 & m & 1-m \\ (1-t)(1-q) & (1-t)q & n & 1-n & 0 \\ tx & t(1-x) & 0 & 0 & 0 \\ ty & t(1-x) & 0 & 0 & 0 \\ 0 & t & 0 & 0 & 0 \end{bmatrix}$$



site
H
D'
1
2
1
2
3
4
5
6
1
2
3
A
status

$$t = p = q = m = n = x = y = 0.5$$

$$a = b = c = d = e = f = 1/3$$

High	Low	Normal	Alert	Failed

$$E[s^T(k)] = [0.4444 \quad 0.5556 \quad 0.2778 \quad 0.5 \quad 0.2222]$$

The probability that, in the next step, the power plan will be in normal status is 0.2778

Planned activities

1) Compute influence matrix for several infrastructures, and apply influence model within an infrastructure and between infrastructures

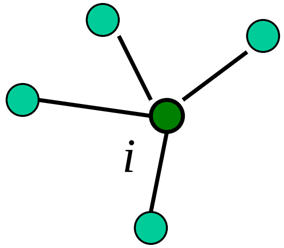
2) How does spread of failures depend on the network topology? How does the behavior of the evil-rain model depend on the network topology?

3) Given a graph, how to compute the influence matrix?

4) Optimization problem for infrastructures

Node degree, average path length, clustering coefficient, betweenness centrality

1/(Node degree)



k_i=4

¼=0.25

$$A=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\1&1&1&1&0\end{bmatrix}$$

$$D=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0.25&0.25&0.25&0.25&0\end{bmatrix}$$

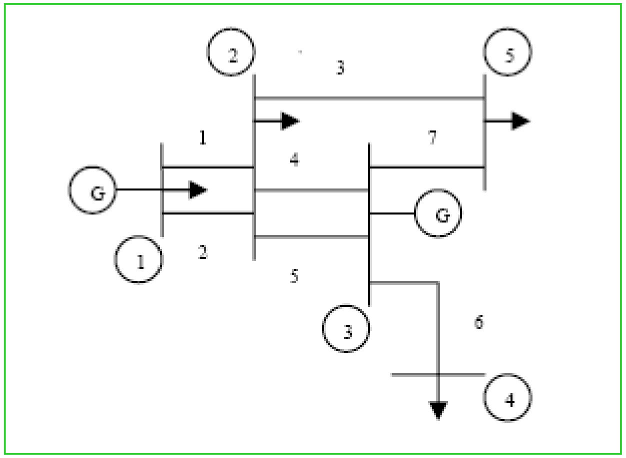
Node (edge) betweenness centrality

$$b_i = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}} ,$$

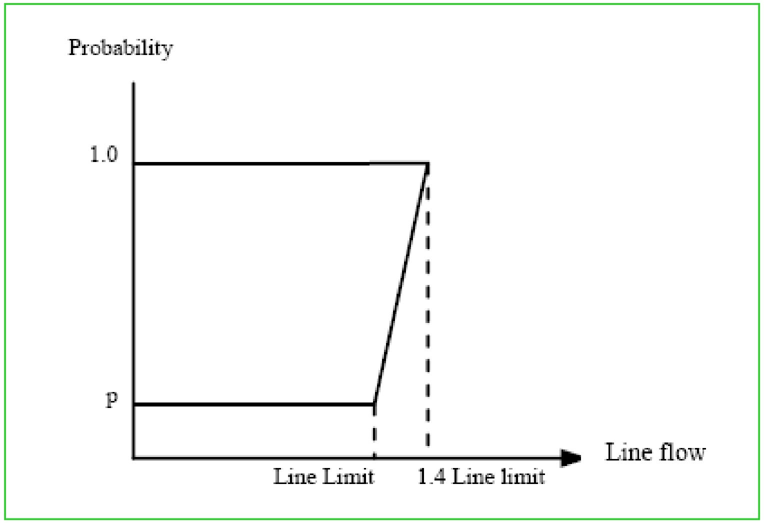
$$b_{i,j} = \sum_{s \neq i \neq j \neq t} \frac{\sigma_{st}(i,j)}{\sigma_{st}} ,$$

$\sigma_{st} = \sigma_{ts}$ is the number of shortest paths between the vertices s and t

$\sigma_{st}(i) = \sigma_{ts}(i)$ is the number of shortest paths going through the node i



5-bus test system



1) For line *i* determine its neighbors by finding all lines *j*, that share bus with it

2) Determine the influence of line *j* on line *i* by removing line *j* from the circuit and run a load flow to measure the percentage overload induced in *i* as a result of the removal of line *j*

3) For a given hidden failure probability *p* and the percentage overload obtained in 2), estimate the probability of exposed line *i* tripping incorrectly (from J.Chen, J.S Thorp, and I. Dobson, "Cascading Dynamics and Mitigation Assessment in Power System Disturbances via a Hidden Failure Model," International Journal of Electrical Power and Energy Systems, 2003.)

4) Repeat steps 1) to 3) for all sites

5) Use the estimated probabilities to form the network influence matrix

• ***Infrastructures as optimizers***

• Communication network: a network that supports a set of flows, each of which has a nonnegative flow rate, and an associated utility function. Each flow passes over a route, which is a subset of the edges of the network. Each edge has a given capacity, which is the maximum total traffic it can support. The ***network utility maximization*** (NUM) problem is to choose the flow rates to maximize the total utility, while respecting the edge capacity constraints.

• Layering as Optimization Decomposition (***networks as optimizers*** and ***layering as decomposition***)