WP6 Vulnerability of interconnected Networks

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Venue: LIUC University

Castellanza - June 28 – 29, Italy

What are Networks?

Interacting many "particle" systems where the interactions are propagated through a discrete structure, ***a graph*** (not a continuum).

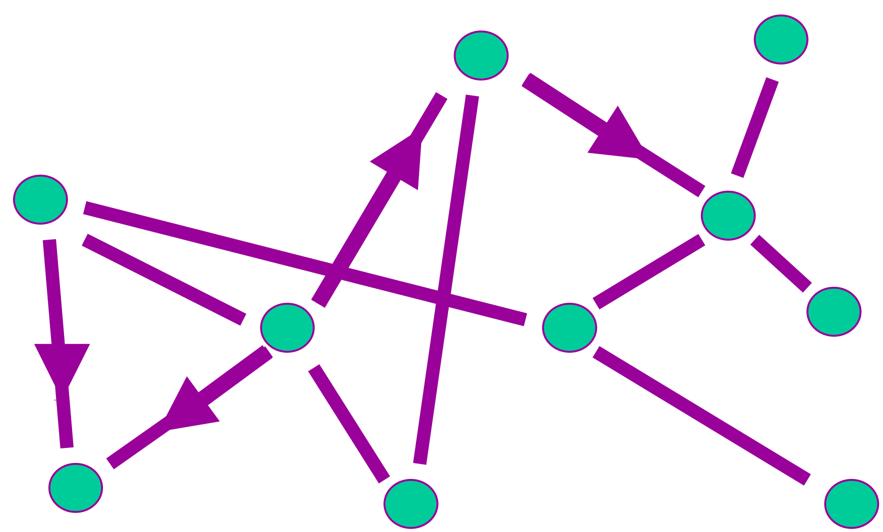
Node (the "particle") Link (edge)

The links *[edges]* represent *interactions or associations* between the nodes.

Graph:

-- undirected

-- directed



Where are Networks?

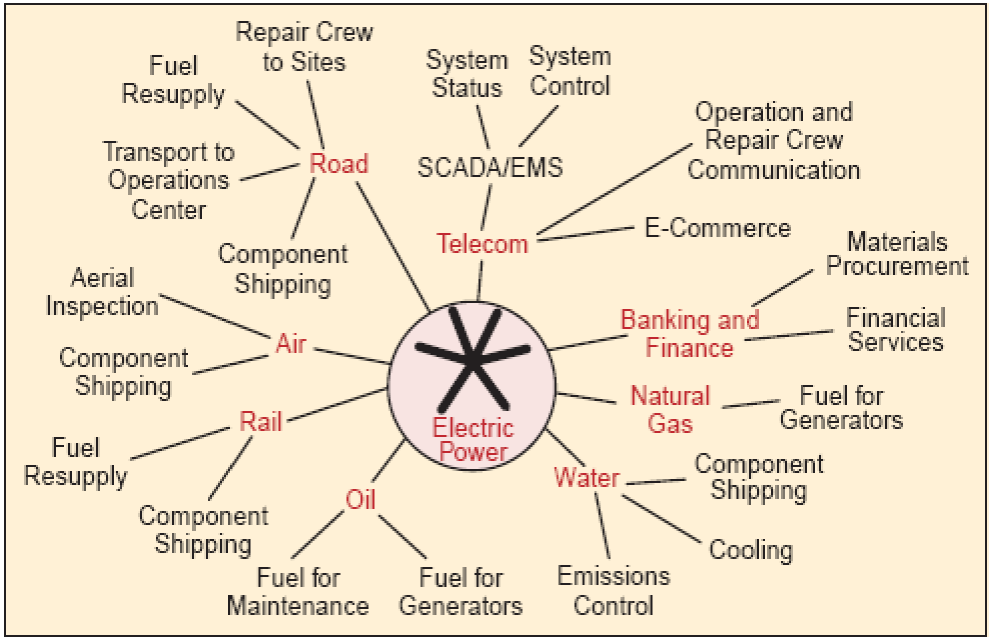
- **Infrastructures:** transportation nw-s (airports, highways, roads, rail, water) energy transport nw-s (electric power, petroleum, natural gas)
- **Communications:** telephone, microwave backbone, internet, email, www, etc.
- **Biology:** protein-gene interactions, protein-protein interactions, metabolic nw-s, cell-signaling nw-s, the food web, etc.
- **Social Systems:** acquaintance (friendship) nw-s, terrorist nw-s, collaboration networks, epidemic networks, the sex-web
- **Geology:** river networks

What is Infrastructure? A network of independent, mostly privately-owned, man-made systems and processes that function collaboratively and synergistically to produce and distribute a continuous flow of essential goods and services.

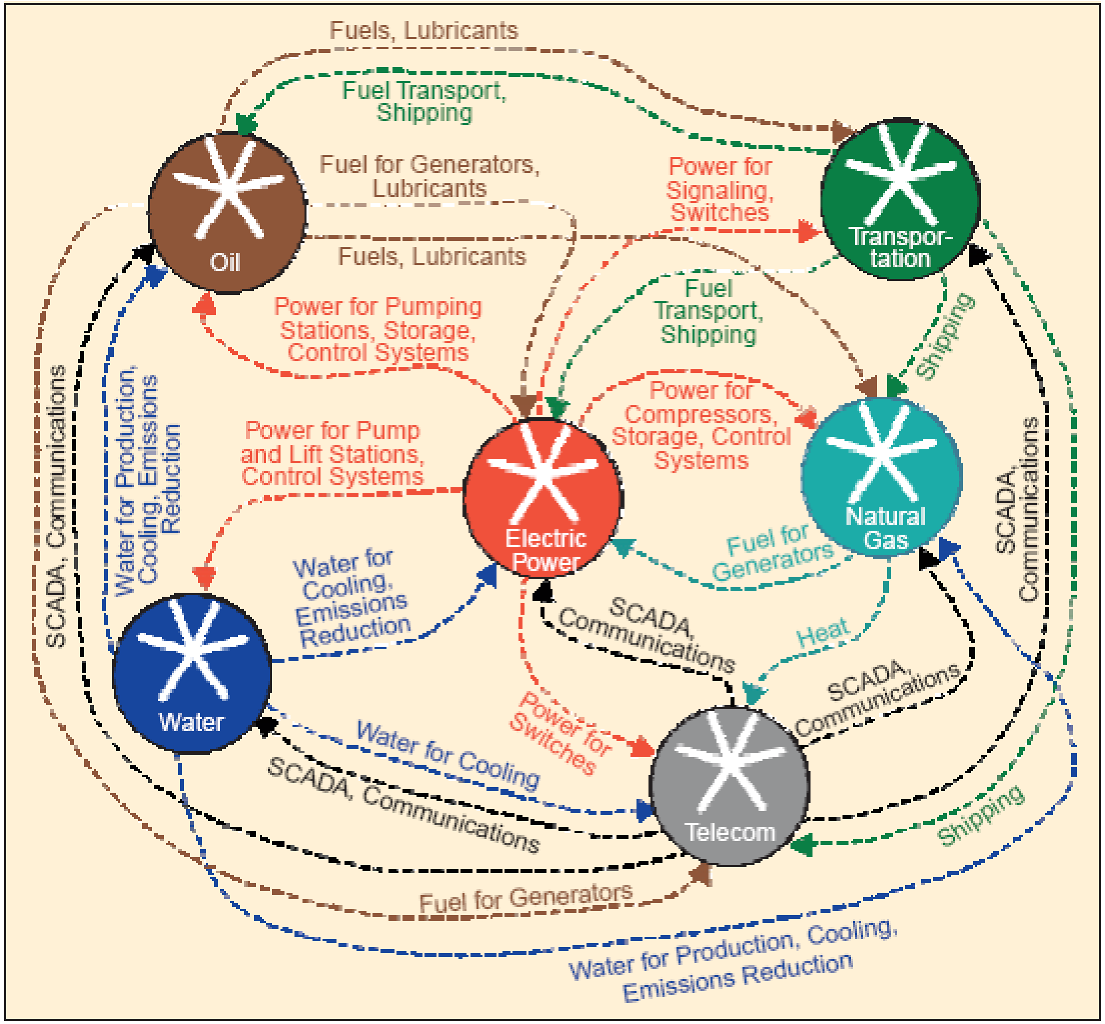
[From: President's Commission on Critical Infrastructure Protection, Critical Foundations: Protecting America's Infrastructures (1997). Available: http://www.ciao.gov]

Eight infrastructures: telecommunications, electric power systems, natural gas and oil, banking and finance, transportation, water supply systems, government services, and emergency services.

Interdependency: A bidirectional relationship between two infrastructures through which the state of each infrastructure influences or is correlated to the state of the other.



Under normal operating conditions, the electric power infrastructure requires natural gas and petroleum fuels for its generators, road and rail transportation and pipelines to supply fuels to the generators, air transportation for aerial inspection of transmission lines, water for cooling and emissions control, banking and finance for fuel purchases and other financial services, and telecommunications for e-commerce and for monitoring system status and system control (i.e., supervisory control and data acquisition (SCADA) systems and energy management systems (EMSs)).

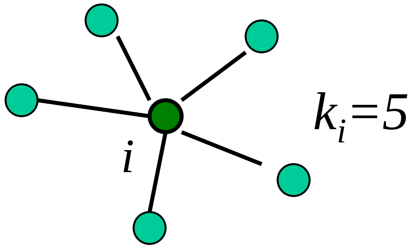


Fuels, Lubricants
Fuel Transport, Shipping
Fuel for Generators, Lubricants
Fuels, Lubricants
Power for Signaling, Switches
Oil
Transpor-tation
Fuel Transport, Shipping
Power for Pumping Stations, Storage, Control Systems
Shipping
SCADA, Communications
Water for Production, Cooling, Emissions Reduction
Power for Compressors, Storage, Control Systems
Power for Pump and Lift Stations, Control Systems
Electric Power
Natural Gas
SCADA, Communications
Fuel for Generators
Water for Cooling, Emissions Reduction
SCADA, Communications
Heat
Water
SCADA, Communications
Power for Switches
Water for Cooling
Telecom
SCADA, Communications
Shipping
Fuel for Generators
Water for Production, Cooling, Emissions Reduction

Structural properties: the scale-free characte

Structural Characterization: fairly well developed for **local network properties** such as degree and clustering, and less well for developed for **global properties** such as shortest path distributions, etc.

Node degree: number of neighbors



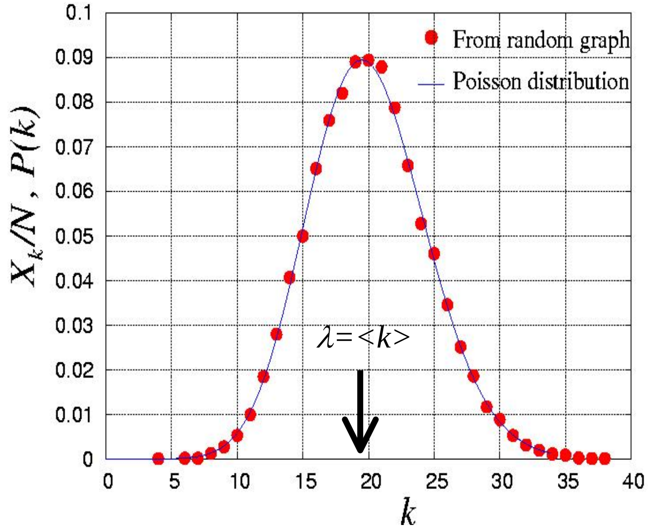
Degree distribution, P(k): fraction of nodes whose degree is k (a histogram over the k_i –s.)

Observation: networks found in Nature and human made, are in many cases "scale-free" networks:

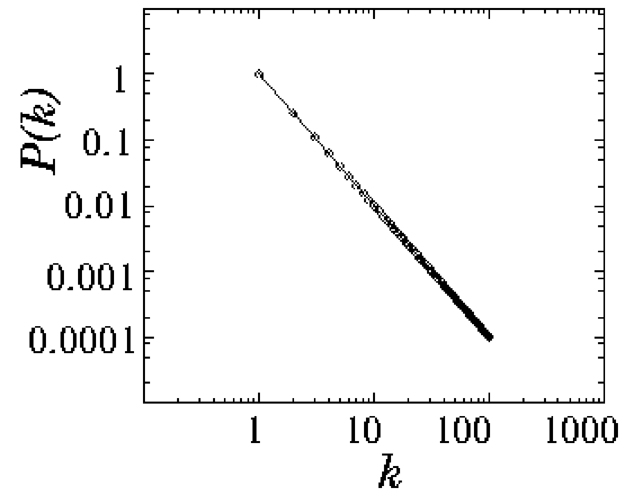
$$P(k) \propto k^{-\gamma}$$

What is scale-free?

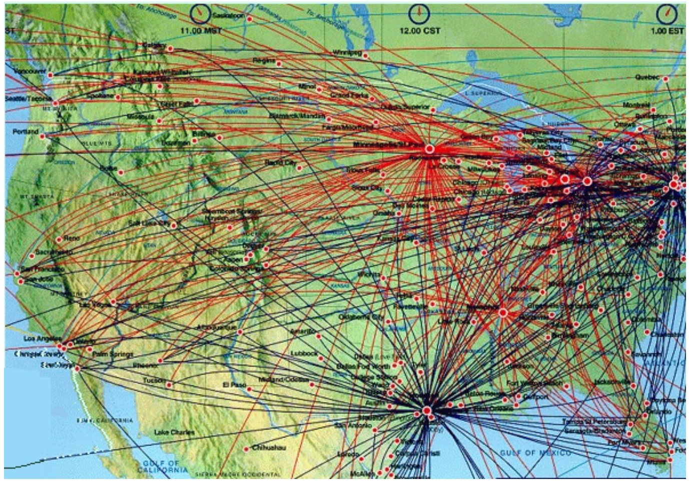
Poisson distribution



Power-law distribution



Non-Scale-free Network



Scale-free Network

Mean shortest distance between vertex pairs in a network:

$$\ell = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}$$

"Harmonic mean" shortest distance between all pairs:

$$\ell^{-1} = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}^{-1}$$

Clustering coefficient

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

$$C = \frac{1}{n} \sum_i C_i$$

What is vulnerability?

There are different approaches to the concept of vulnerability.

One trend relates the vulnerability or robustness of a network with its connectivity, while others relate it with the decrease of efficiency when some vertices or edges are under attack.

DYNAMIC RESPONSE OF NETWORKS

Dynamic responses are captured by two main properties:

(1) network resilience, and
(2) network fragmentation

Network resilience and fragmentation are essential to describe the performance of a network when its components are being removed, and to identify whether or not its basic topological properties influence the response.

Network Resilience

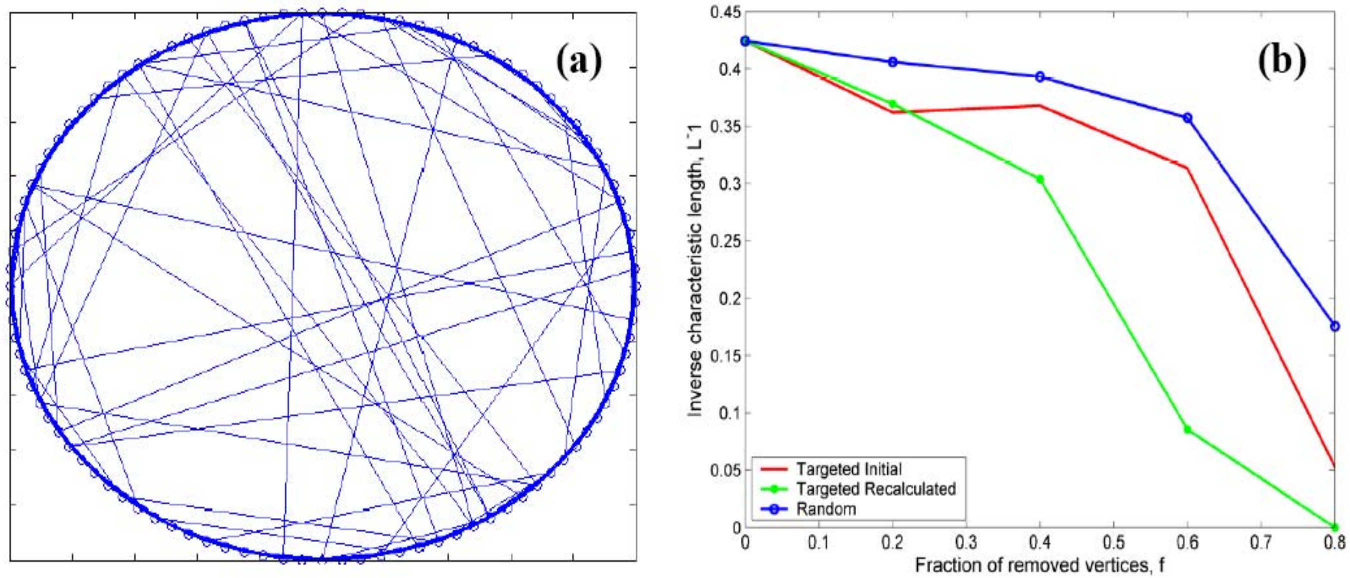
Network resilience: property that characterized its capacity to remain connected after vertex removals; they are performed incrementally until the fraction of removed vertices within a network of order, n, approaches 1.

Different strategies are followed for systematic removal:

- Random removal

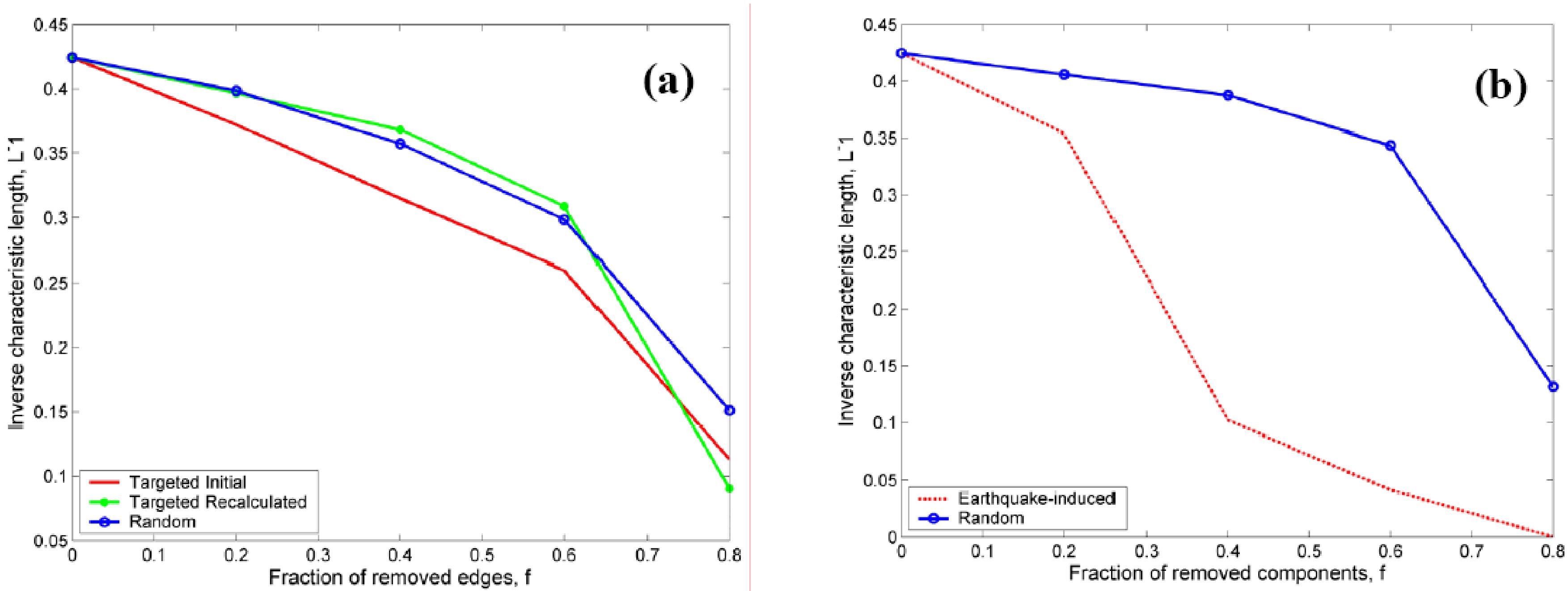
- Deliberate attacks: selects vertices or edges based on their degree, and preferentially removes the most connected ones

In the context of natural hazards of large extent (e.g., earthquakes), the removal strategy corresponds to neither vertex deletion, nor to edge deletion alone. <u>It involves simultaneous removal of vertices and edges</u>, selecting components by following a relationship between earthquake hazard intensity and structural response.



(a) Small-world network model of order n = 100, mean vertex degree, k = 10, and rewiring probability β = 0.01.

(b) Network model resilience to vertex removal.



(a) Network model resilience to edge removal
(b) Network model resilience to simultaneous vertex and edge removal
(e.g., earthquake-induced hazard)

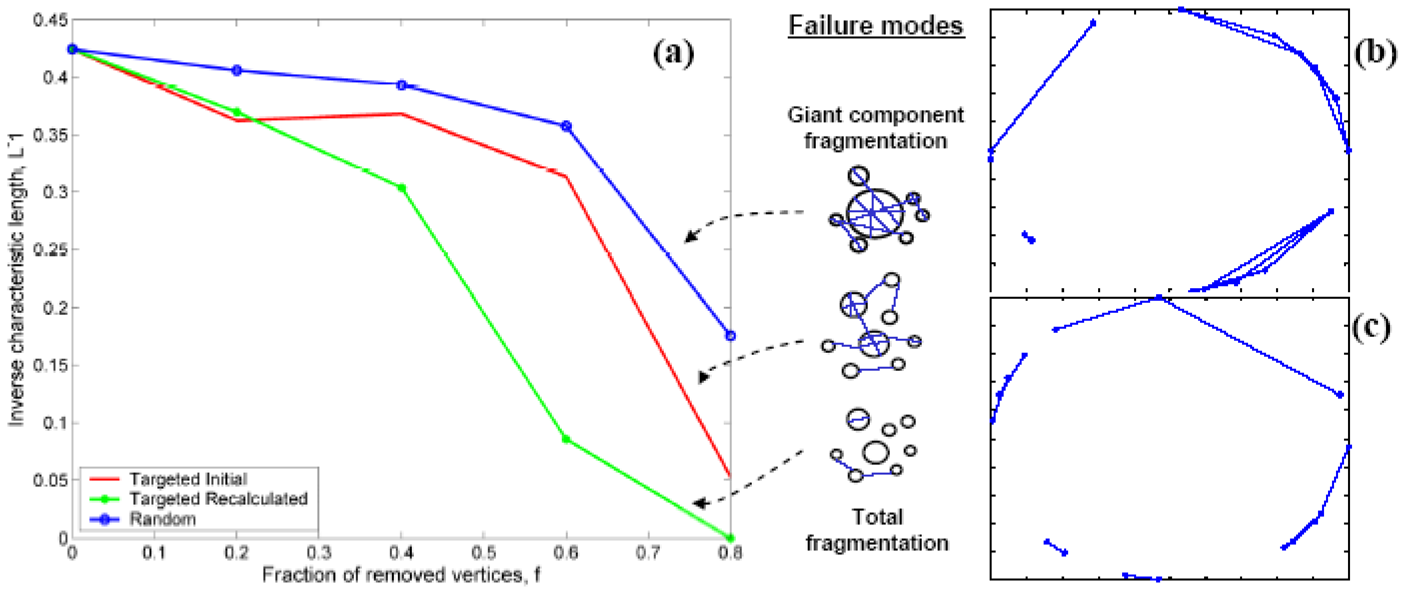
Network fragmentation

Fragmentation captures the number and size of the portions of the network that become disconnected.

There are two extreme failure mechanisms:

(1) giant component fragmentation, and

(2) total fragmentation.



(a) Association of failure modes with network resilience.

(b) and (c) Remaining network components of small-world model after 80% random and targeted initial removals.

Vulnerability function

DEFINITION 2.1. Let $\mathcal{G}$ be the set of all possible graphs with a finite number of vertices. A *vulnerability function* v is a function $v: \mathcal{G} \longrightarrow [0, 1]$ verifying the following properties:

(V1) v is invariant under isomorphisms.
(V2) $v(G') \geqslant v(G)$ if G is obtained from G' by adding edges.
(V3) $v(G)$ is computable in polynomial time respect to the number of vertices of G.

What is vulnerability?

INTERDEPENDENT INFRASTRUCTURE VULNERABILITY

1. Yacov Y. Haimes and Pu Jiang: “LEONTIEF-BASED MODEL OF RISK IN COMPLEX INTERCONNECTED INFRASTRUCTURES”, Journal of Infrastructure Systems, Vol. 7, No. 1, March, 2001.

2. Yacov Y. Haimes; Barry M. Horowitz; James H. Lambert; Joost R. Santos; Chenyang Lian; and Kenneth G. Crowther: “Inoperability Input-Output Model for Interdependent Infrastructure Sectors. I: Theory and Methodology”, Journal of Infrastructure Systems, Vol. 11, No. 2, June 1, 2005.

3. Yacov Y. Haimes; Barry M. Horowitz; James H. Lambert; Joost Santos; Kenneth Crowther; and Chenyang Lian: “Inoperability Input-Output Model for Interdependent Infrastructure Sectors. II: Case Studies”, Journal of Infrastructure Systems, Vol. 11, No. 2, June 1, 2005.

LEONTIEF INPUT-OUTPUT MODEL

Original Leontief input-output (I/O) model is a framework for studying the equilibrium behavior of an economy.
The economy (system) is assumed to consist of a group of n interacting sectors or industries, where each ‘‘industry’’ produces one product (commodity). A given industry requires labor, input from the outside, and also goods from interacting industries. Each industry must produce enough goods to meet both interacting demands (from other industries in the group) plus external demands (e.g., foreign trade and industries outside the group).

x_j output of the jth goods $j = 1,2,...,n$

r_j input of the ith resource $i = 1,2,...,m$

x_{kj} amount of the kth goods used in the production of the jth goods

r_{ij} amount of the ith resource input used in the production of the jth goods

Leontief's model assumes that the inputs of both goods and resources required to produce any commodity are proportional to the output of that commodity

$$x_{kj} = a_{kj} x_j \quad k, j = 1,2,\ldots,n$$

$$r_{ij} = b_{ij} x_j \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n$$

Furthermore, the output of any commodity is used either as input for the production of other commodities or as final demands, c_k

$$x_k = \sum x_{kj} + c_k \quad k = 1,2,\ldots,n$$

$$\boxed{x_k = \sum_j a_{kj} x_j + c_k \quad k = 1,2,\ldots,n}$$

LEONTIEF-BASED INPUT-OUTPUT INFRASTRUCTURE MODEL

Inoperability of a system is defined as the inability of the system to perform its intended functions: it is assumed to be a continuous variable between 0 and 1 (0 = flawless operable system, 1 = system being completely inoperable)

c – input to the interconnected infrastructures: perturbations in the form of natural events, accidents, or willful attacks
x – vector of inoperability of the different infrastructures due to their connections to the perturbed infrastructure and to one another

The long-run inoperabilities of the interconnected infrastructures can be calculated using the following Leontief-type equation:

$$x_k = \sum_j a_{kj} x_j + c_k \quad k = 1,2,\ldots,n$$

a_{kj} – probability of inoperability that the *j*th infrastructure contributes to the *k*th infrastructure due to the complexity of their interconnectedness.

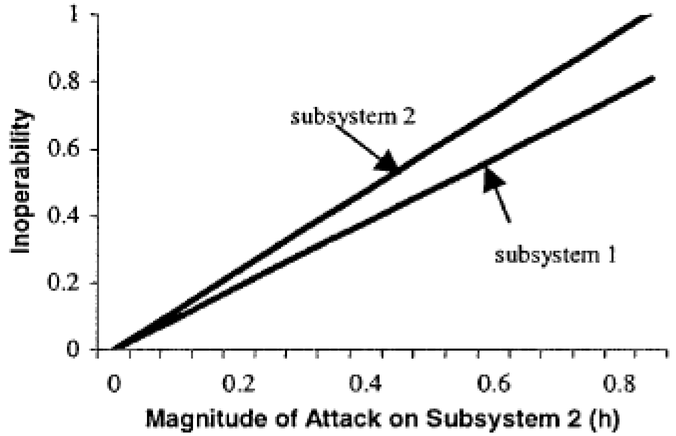
Example 1: Suppose we have a system with two subsystems. Further suppose a failure at subsystem 2 will lead subsystem 1 to be 80% inoperable, and a failure at subsystem 1 will lead subsystem 2 to be 20% inoperable.

$$A = \begin{pmatrix} 0 & 0.8 \\ 0.2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 0.8 \\ 0.2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ h \end{pmatrix}$$

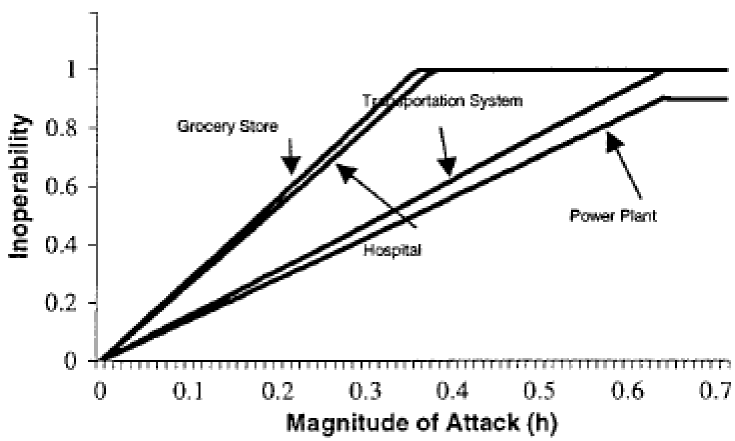
$$x_1 = 0.952h \qquad x_2 = 1.190h \quad \text{for } 0 \le h \le 0.84$$

$$x_1 = 0.8 \qquad x_2 = 1 \quad \text{for } 0.84 \le h \le 1$$



Example 2:
Subsystem 1: A power plant
Subsystem 2: A transportation system
Subsystem 3: A hospital
Subsystem 4: A grocery store

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 0.9 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 1 & 0.8 & 0 & 0 \\ 1 & 0.9 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ h \\ 0 \\ 0 \end{pmatrix}$$



If the power plant fails completely, then the transportation system can perform only 60% of its functionality, whereas both the hospital and the grocery store cannot operate at all.

If the transportation system completely fails, then the power plant and the grocery store can perform only 10% of their full functionality, and the hospital can perform only 20% of its functionality.

The inoperability of the hospital or the grocery store has no impact on the operation of the power plant or the transportation system, nor do they have any appreciable impact on each other.

Now, suppose a major hurricane hits the area and destroys *h* (50%) of the functionality of the transportation system.

Conclusions and future work

- Network resilience
- Network fragmentation
- Infrastructure vulnerability

- Different methods for calculation of the matrix A
- Nonlinear and dynamic models and effects
- Topological effects on the matrix A