A Model for Information Spreading over Networks

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A novel model for information (rumor) spreading over networks is suggested, in which two different kinds of information, one of which is always considered first, may propagate among the nodes. The model is a natural generalization of the well-known epidemic SIS (susceptible-infective-susceptible) model, and reduces to it when some of the parameters of this new model are zero. We find that the preferred information type 1 is heavily promoted when the degree of nodes is high enough and/or when the network contains large clustered groups of nodes, expelling information type 2. In particular, the information type 1 is dominant when the average degree in Erdos-Renyi graphs is increased, the cliquish neighborhoods in Watts-Strogatz small-world networks are increased, and the number of links m, when a new node is added, in Barabasi-Albert networks is increased. On the other hand, simulations show that it is possible for information type 2 to occupy a nonzero fraction of the nodes in many cases as well. Specifically, in the Watts-Strogatz small-world model some level of clustering facilitates its adoption, while increasing randomness undermines it. For Erdos-Renyi networks, a low average degree allows the coexistence of the two types of information. In Barabasi-Albert networks generated with a small m, it is also possible for information type 2 to spread over the network.

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I. INTRODUCTION

The investigation of social spreading phenomena such as the propagation of rumors, the diffusion of fads, the adoption of technological innovations, and the success of consumer products mediated by word-of-mouth, has a long tradition in sociology and economics. Effects of the network of contacts in the spreading process have been postulated long since ([1, 2]), and recently, with the development of the theory of complex networks, these effects are gradually unravelled. In fact, in the last decade, complex networks theory has paved the way for exploring many real-world large-scale networks, and describing and understanding various processes that play out on the nontrivial topology of these networks. Much of the studies in this respect are concerned with spreading processes such as virus propagation in social and computer networks ([3–8]), the diffusion of innovations [9, 10], the occurrence of information cascades in social and economic systems [11], disaster spreading in infrastructures [12], or information diffusion in a society through the word-of-mouth (w-o-m) mechanism [13], to name a few.

However, in the context of complex networks research, so far the spread of only one type of influence through a network has been considered (a notable exception is [14], as we outline later). In this paper we present a model of information (rumor) spreading, or influence diffusion through a network, where two different types of information affect the nodes, and consider the behavior of the model for different network topologies.

Ones of the earliest and widely used models for describing collective social behavior are the threshold models, first proposed by Granovetter in [15]. Each individual has a specific threshold value, based on which a binary decision is made. More formal definitions which take social network structure into account have appeared since, the simplest version of which is the linear threshold model [16]. A variant of the threshold model has been used, for example, in [9, 10] for describing diffusion of innovations in a population.

The model we propose does not involve a threshold value for an individual: hearing information from a single neighbor is generally enough for an agent to start spreading it. It is then clear that information spreading bears a lot of resemblance to the evolution of an epidemic, with informed people playing the role of infected agents and uninformed people that of susceptible ones. Indeed, the most popular model for information or rumor spreading, introduced by Daley and Kendall [17], see also [18–21], is conceptually similar to the SIR model for epidemiology. Agents are divided into three classes: ignorants, spreaders, and stiflers, i.e., those who have lost interest in diffusing the information or rumor. Their role is exactly the same as the susceptible, infected, and recovered agents of the SIR model. Other uses of epidemiological models for describing information spread are, for example, topic flow in blogspace [22], and word-of-mouth in product marketing [13].

The model we propose is similar in nature to the SIS model, where a node in the network can be in the *susceptible* state, not having contracted the disease, or in the *infective* state, able to spread the disease to each of its neighbors. Infectives recover, becoming susceptible to the disease again. In our model the nodes are divided into three classes: ignorants, class-one spreaders, and class-two spreaders. Namely, each node is "susceptible" to the effects of two kinds of "infections", and is able to recover from them as well. Moreover, one of the information sources has a dominant position, in the sense that the nodes will always consider information coming from this node first. This formulation could be significant for describing two w-o-m processes circulating in a social network, e.g. concerning two competing products in the market, or two candidates form opposing parties in elections, one of which has a better standing in the public eye. We note that information spreading also has appealing connections with the search for robust scalable communication protocols in large distributed systems [23, 24], "viral" strategies in marketing [25], and epidemic routing in ad hoc networks [26, 27].

When considering information spreading, some of the relevant questions, which will be addressed in this paper, are similar to those for epidemiology: How many people will eventually be reached by the news? Is there an "epidemic threshold" for the rate of spreading, separating a regime in which a finite fraction of people will be informed from one with the information remaining confined to a small neighborhood? There is a large amount of studies concerning epidemic spreading from a complex networks perspective. Using percolation theory ideas and generating function methods, [3] gives exact analytical results for the epidemic threshold, outbreak size and other relevant quantities for the SIR model. The results represent average values over an ensemble of random graphs with an arbitrary degree distribution. In [4, 5], the absence of an epidemic threshold has been established for the SIS model and infinitely large networks with a power-law degree distribution $k^{-\gamma}$, $2 < \gamma \leq 3$, whereas [6] show the existence of a threshold for clustered power-law networks. Rather than determining the epidemic threshold for a whole class of networks with a given degree distribution, [7] and [8] propose its calculation for a specific network given with an adjacency matrix, for a SIR and SIS model, respectively. We follow this idea to determine the information spreading threshold in our model.

Finally, we outline a study which is close to our work in the sense that two information types spread through the network. Namely, in [14], Goldenberg et al. investigate the effects of both positive and negative w-o-m on a firm's profits. In this study, negative w-o-m is limited to travelling two hops away from its source. In our model, both information types can spread arbitrarily far from their respective sources.

The paper proceeds as follows. Section II defines the model and analyzes the stability of its dynamics. In Section III we describe the behavior of the model on regular network topologies, such as the cycle, star, and fully connected topology. Results of the information dissemination simulations on complex network topologies are given in Section IV. The last section concludes the article and points out potential research directions.

II. THE INFORMATION SPREADING MODEL

A. Definition of the model

Consider a closed population of N individuals, connected by a neighborhood structure which is represented by an undirected, unweighted graph $G = (V, E)$ with node set V and edge set E. Let A denote the adjacency matrix of the graph G, i.e., $a_{ij} = 1$ if $(i, j) \in E$ and $a_{ij} = 0$ otherwise. We propose a discrete stochastic model for information spreading in a network. The model allows for two different types of information, or influences, coming from two different sources, to spread through the network. At time k, each node i can be in one of three possible states: 1, 2 and 3. The state of the node is indicated by a status vector, an indicator vector containing a single 1 in the position corresponding to the present state, and 0 everywhere else:

$$\mathbf{s}_i(k) = [s_i^1(k) \;\; s_i^2(k) \;\; s_i^3(k)]^T,$$

for all $i \in 1, ..., N$. A node being in state 1 or 2 indicates that it is a supporter, or adopter of the information of type 1 or 2, accordingly. State 3 signifies an undecided, or neutral state of the node in relation to the two types of information circulating in the network. Let

$$\mathbf{p}_i(k) = [p_i^1(k) \;\; p_i^2(k) \;\; p_i^3(k)]^T$$

be the probability mass function of node i at time k. For every node i it states the probability of being in each of the possible states at time k. Having defined that, the equations describing the dynamics of each node, i.e. the evolution of the model, are:

$$\begin{array}{rcl} p_i^1(k+1) & = & s_i^3(k) f_i + a_1 s_i^1(k) \\ p_i^2(k+1) & = & s_i^3(k)(1-f_i) g_i + a_2 s_i^2(k) \\ p_i^3(k+1) & = & s_i^3(k)(1-f_i)(1-g_i) + \\ & & +(1-a_1) s_i^1(k) + (1-a_2) s_i^2(k) \\ \mathbf{s}_i^T(k+1) & = & MultiRealize[\mathbf{p}_i^T(k+1)] \end{array} \quad (1)$$

where $MultiRealize[\cdot]$ performs a random realization for the probability distribution given with $\mathbf{p}_i^T(k+1)$. a_1 and a_2 are parameters, $0 \leq a_1 \leq 1$ and $0 \leq a_2 \leq 1$. In the model f_i and g_i are given by:

$$\begin{array}{l} f_i(k) = 1 - \prod_{j=1}^{N} (1 - \beta a_{ij} s_j^1(k)) \\ g_i(k) = 1 - \prod_{j=1}^{N} (1 - \gamma a_{ij} s_j^2(k)) \end{array} \quad (2)$$

In the last two expressions β and γ are parameters, $0 \leq \beta \leq 1$ and $0 \leq \gamma \leq 1$. We note that a discrete stochastic SIS model can be obtained as a special case of the information spreading model by setting $a_2 = 0, \gamma = 0$ and no nodes in status 2 initially. Status 1 is then the infective state, and status 3 is the susceptible state, with the curing rate of the disease being $\delta = 1 - a_1$.

The mechanism of spreading is the following. Each node in status 1 attempts to send a message to each of its neighbors at the beginning of time k. Each attempt is successful with probability β and is independent of other attempts. The nodes in status 2 also send messages to each of their neighbors with probability γ. Hence, f_i and g_i are the probabilities that node i receives a status 1 or status 2 message from its neighboring supporters of information types 1 and 2, accordingly. However, note from the formulation of (1) that node i will actually be able to become a supporter of information 2 at $(k+1)$ only if, at time k, it does not receive a status 1 message from its neighbors. More precisely, the probability of converting from an undecided status to status 2 is not g_i, but it is multiplied by the factor $(1 - f_i)$ which in general is smaller than 1. Conversely, node i can become a supporter of information 1 regardless of whether it has received a status 2 message or not. In this way, the model has prebuilt a preference in each node for the information of type 1, as if though its source was more credible or more reputable. Additionally, after adopting a particular type of information of influence, the nodes in states 1 and 2 continue to preserve their status at a rate of a_1 and a_2, respectively, i.e. they convert back to status 3 with a rate of $1 - a_1$ and $1 - a_2$, respectively. The parameters a_1 and a_2 can be said to signify the remembrance rate of each type of information, or how long the nodes are willing to support the adopted type of information before abandoning it and converting back to an undecided status.

Let us now elaborate on the relation of the model to some real-world examples. For marketing purposes, we can consider a situation where a new product is entering the market, having to compete with a product of the same kind from an already established brand-name company. As for the situation of elections, one could take the case of a country which has a dominant political party, and each citizen is inclined to consider a candidate from this party first. However, in these real-world examples the decision of an individual is influenced not just by the decisions of their peers, but by factors such as mass-media advertising, or political campaigns. These factors have an effect on every person in the population, and are not encompassed in the model. Since in the model information spreads through the interaction of individuals, it is safe to say that it simulates the spreading of rumor or w-o-m about two different products, or two candidates from opposing political parties. It can be useful to investigate how w-o-m affects the adoption of a particular attitude by the consumers or voters. Furthermore, the model makes a simplifying assumption that the remembrance rates a_1 and a_2 and the message transmission probabilities β and γ are the same for all nodes. In a real-world scenario where each node represents an individual, they would depend on one's characteristics and preferences.

Let $X(k) = \sum_{i=1}^{N} s_i^1(k)$, $Y(k) = \sum_{i=1}^{N} s_i^2(k)$ and $Z(k) = \sum_{i=1}^{N} s_i^3(k)$ be the total number of nodes in statuses 1, 2 and 3 at time k, respectively. Further, let $N_1 = \mathbb{E}[X(\infty)]$, $N_2 = \mathbb{E}[Y(\infty)]$ and $N_3 = \mathbb{E}[Z(\infty)]$. The object of interest is the average number of nodes that eventually (when $k \to \infty$) become in statuses 1 and 2, N_1 and N_2, compared to the total number of nodes N in the network.

In the following, in order to facilitate the mathematical analysis we rewrite the model given with (1) and (2) as:

$$\begin{aligned} p_i^1(k+1) &= p_i^3(k) f_i + a_1 p_i^1(k) \\ p_i^2(k+1) &= p_i^3(k)(1-f_i) g_i + a_2 p_i^2(k) \\ p_i^3(k+1) &= p_i^3(k)(1-f_i)(1-g_i) + \\ &\quad (1-a_1) p_i^1(k) + (1-a_2) p_i^2(k) \end{aligned} \quad (3)$$

and f_i and g_i as:

$$\begin{aligned} f_i(k) &= 1 - \prod_{j=1}^{N} (1 - \beta a_{ij} p_j^1(k)) \\ g_i(k) &= 1 - \prod_{j=1}^{N} (1 - \gamma a_{ij} p_j^2(k)) \end{aligned} \quad (4)$$

Equivalently N_1, N_2, N_3 can be computed using Eq. (3) as $N_1 = \sum_{i=1}^{N} p_i^1(\infty)$, $N_2 = \sum_{i=1}^{N} p_i^2(\infty)$, and $N_3 = \sum_{i=1}^{N} p_i^3(\infty)$.

To illustrate that (1) and (3) are the same, consider Fig. (1). The figure shows the results of running both of the models on a 30 node scale-free network generated by the Barabasi-Albert (BA) algorithm as given in [28], with $m = 2$. In this case we simulate injecting the two information types at two sources, meaning that they are allowed to change their initial status as time passes. As can be seen, the evolution of the sum probability vector according to (3), corresponds to the evolution of the number of nodes in each status as predicted by (1). In effect, by studying (3), we are investigating the evolution of the probability vectors from which random realizations are made in (1).

B. Dynamical systems approach

In this part we apply a dynamical systems approach to our model. Let us replace the probabilities for the node i to be in status 1 and 2 with $x_i = p_i^1$ and $y_i = p_i^2$, respectively, in (3). The evolution of the model can be rewritten as:

$$\begin{aligned} x_i(k+1) &= [1 - x_i(k) - y_i(k)] f_i(k) + a_1 x_i(k) \\ y_i(k+1) &= [1 - x_i(k) - y_i(k)][1 - f_i(k)] g_i(k) + a_2 y_i(k), \end{aligned} \quad (5)$$

where

$$\begin{aligned} f_i(k) &= 1 - \prod_{j=1}^{N} (1 - \beta a_{ij} x_j(k)) \\ g_i(k) &= 1 - \prod_{j=1}^{N} (1 - \gamma a_{ij} y_j(k)). \end{aligned} \quad (6)$$

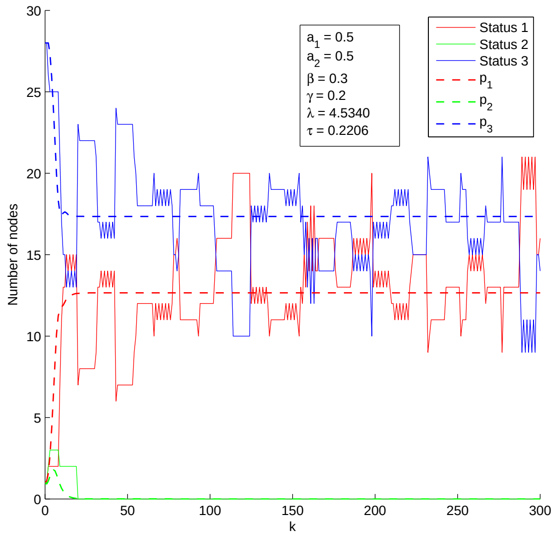


FIG. 1: Evolution of (1) and (3) on a 30 node BA network generated with $m = 2$. Initially, node 25 is a supporter of information type 1, while node 29 of information type 2. Both nodes have degree 2, and may change their initial status as time progresses. The solid lines show the number of nodes in each status as simulated by (1), while the dashed lines show the average number of nodes in each status as obtained by $N_1 = \sum_{i=1}^{N} p_i^1(\infty)$, $N_2 = \sum_{i=1}^{N} p_i^2(\infty)$, and $N_3 = \sum_{i=1}^{N} p_i^3(\infty)$.

Equation (5) represents a nonlinear dynamical system $F : [0,1]^{2N} \to [0,1]^{2N}$. Since x_i and y_i are probabilities, and assuming that the corresponding graph is connected, then the ergodicity of the Markov chain of the whole system is guaranteed [29] if

$$a_1 \neq 1, a_2 \neq 1, \beta \neq 0, \beta \neq 1, \gamma \neq 0, \gamma \neq 1. \qquad (7)$$

Therefore, when the condition (7) is satisfied, the dynamical system (5) has a unique globally stable fixed point.

The system (5) has a fixed point at $(x_i, y_i) = (0, 0)$ for all i. The local stability of this fixed point can be analyzed using the Jacobian matrix of the system (5) evaluated at the fixed point,

$$DF|_{(0,0)} = \begin{bmatrix} A_\beta & 0_N \\ 0_N & A_\gamma \end{bmatrix},$$

where $A_\beta = a_1 I + \beta A$, $A_\gamma = a_2 I + \gamma A$, and 0_N (the matrix of all 0 elements) are $N \times N$ matrices. Hence the fixed point $(0, 0)$ is stable when

$$\max\{a_1 + \beta\lambda, a_2 + \gamma\lambda\} < 1, \qquad (8)$$

where λ is the largest eigenvalue of the adjacency matrix. What this condition means with regard to information spread is that, whenever it is fulfilled, eventually no information of either type will persist in the network. All of the nodes will be in the neutral status, since the model stabilizes in a state where the probabilities of each node to adopt either 1 or 2 are zero.

Restating condition (8) as

$$\frac{\beta}{1 - a_1} < \frac{1}{\lambda}, \frac{\gamma}{1 - a_2} < \frac{1}{\lambda} \qquad (9)$$

one can see that the value of $\tau = \frac{1}{\lambda}$ appears as a threshold value for the ratios $\frac{\beta}{1-a_1}$ and $\frac{\gamma}{1-a_2}$ up to which the fixed point $(0, 0)$ is stable. In these ratios, β and γ are the message transmission probabilities. Since we interpreted a_1 and a_2 as the remembrance rates of the two information types, $(1 - a_1)$ and $(1 - a_2)$ are the rates at which the information types dissipate at the nodes. Whenever a ratio of transmission to dissipation is larger than the threshold, the fixed point $(x_i, y_i) = (0, 0)$ is unstable, and there will be information spread in the network. The existence of a network threshold $\frac{1}{\lambda}$ has already been pointed out in several epidemiological models of virus spreading such as the SIS-type model in [7] and the SIR model in [8].

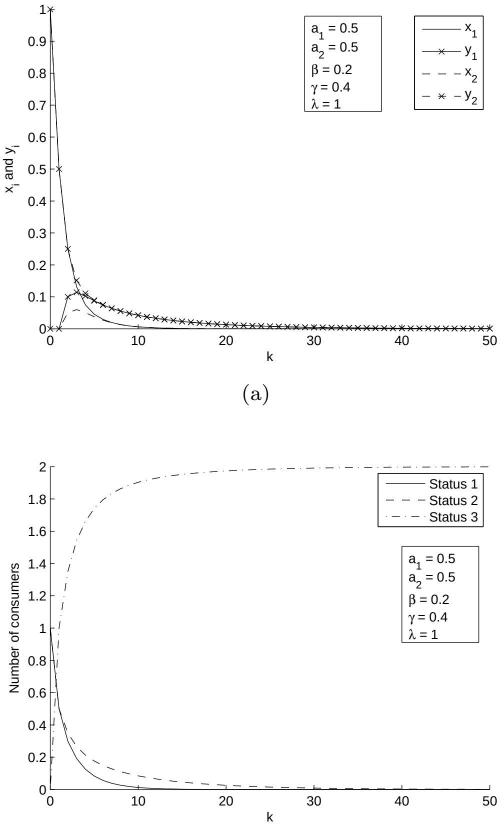
C. Example

Now consider a simple 2-node example. Two consumers are in contact with each other. Each of them has adopted a product and will spread the word about it to the other with probability β or γ. For the two consumers, the evolution equations (5) become:

$$\begin{aligned} x_1(k+1) &= [1 - x_1(k) - y_1(k)]\beta x_2(k) + a_1 x_1(k) \\ x_2(k+1) &= [1 - x_2(k) - y_2(k)]\beta x_1(k) + a_1 x_2(k) \\ y_1(k+1) &= [1 - x_1(k) - y_1(k)][1 - \beta x_2(k)]\gamma y_2(k) + \\ & \quad a_2 y_1(k) \\ y_2(k+1) &= [1 - x_2(k) - y_2(k)][1 - \beta x_1(k)]\gamma y_1(k) + \\ & \quad a_2 y_2(k), \end{aligned} \qquad (10)$$

where x_i and y_i, $i = 1, 2$, are the probabilities that a consumer adopts product 1 or 2, respectively. Since $\lambda = 1$ for a network of 2 nodes, whenever $a_1 + \beta < 1$ and $a_2 + \gamma < 1$, the fixed point $(0, 0)$ is stable. In this case the consumers adopt neither product. One interpretation is that the products are not good enough to be remembered or supported by the consumers (i.e. a_1 and a_2 are not high enough) and/or the consumers do not share their opinions about the products often enough (low message transmission parameters β and γ).

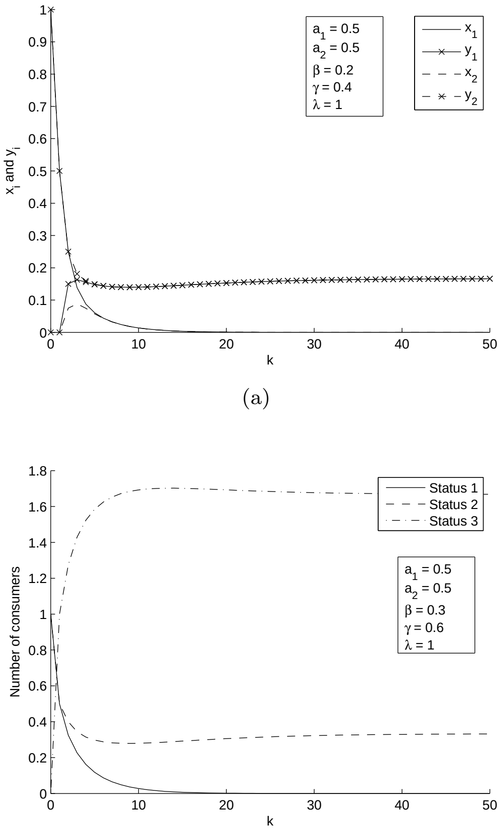
Several numerical experiments are presented to investigate what happens for different values of the parameters. Each experiment starts with consumer 1 having adopted product 1, and consumer 2 supporting product 2. The results show the evolution of (10), and the evolution of the average number of consumers in each status. Fig. (2) illustrates the case when the stability condition for the fixed point $(0, 0)$ is fulfilled.



(a)

(b)

FIG. 2: The evolution of (10). The condition for the stability of $(0, 0)$ is satisfied. (a) Since $\gamma > \beta$, y_i decay more slowly towards 0 than x_i, i.e. the second product experiences a slightly longer usage in the two-consumer network. (b) The average number of consumers in each status.



(a)

(b)

FIG. 3: The evolution of (10). $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$ and $\frac{\beta}{1-a_1} < \frac{1}{\lambda}$. (a) Each of the two consumers has the same nonzero probability to be in status 2 when the system stabilizes, and no probability to be in status 1. (b) The average number of consumers in each status.

Fig. (3) shows that when one of the ratios, in this case the one for product 2, is higher than the threshold value, the system stabilizes in a point where both y_1 and y_2 are nonzero and have the same value. x_1 and x_2 tend to zero values as expected, since the value of $\frac{\beta}{1-a_1}$ is below the threshold. This means that in the end, product 2 has managed to persist in the "network" with the two consumers being equally probable to adopt it. A similar situation occurs when $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$, $\frac{\gamma}{1-a_2} < \frac{1}{\lambda}$ (Fig. (4)).

Let us now consider the case when both products are able to spread through the two-consumer network. Curiously enough, in the case of Fig. (5a) product 2 totally "suppresses" product 1, which at the end of the experiment has almost no probability of occurring at the either of the nodes. Conversely, if we reverse the values of all the parameters as in Fig. (5b), product 1 is the prevailing one. Also, it can be seen that the evolution of the model is not symmetrical in relation to the change in parameters - the convergence to the steady state in Fig. (5b) is faster.

To summarize, for both types of information to be able to spread in a network, it is necessary that their transmission to dissipation rates are greater than the network threshold. However, whether at the end both information types will still persist in the network depends on the particulars of the case, as we discuss later in the article.

III. BEHAVIOR OF THE MODEL ON REGULAR NETWORK TOPOLOGIES

This section presents results on the model behavior on regular networks. The topologies considered are the cycle, star, and fully connected graph. In all the experiments initially there is one node in status 1 and one in status 2. The information is just injected in these two sources, allowing them to change their status as time passes. Also recall that for the convergence of the model to a unique fixed point we established condition (7). It will hold for all the numerical simulations.

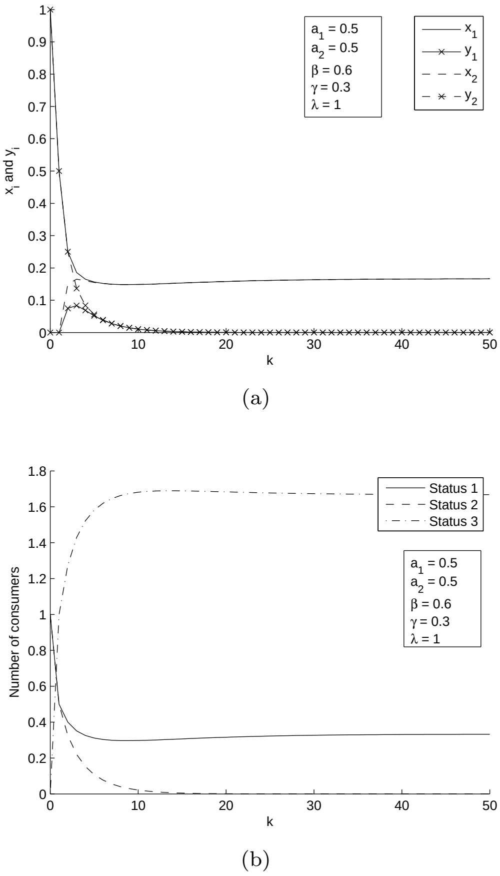


FIG. 4: The evolution of (10). $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} < \frac{1}{\lambda}$. (a) Each of the two consumers has the same nonzero probability to be in status 1, when the system stabilizes, and no probability to be in status 2. (b) The average number of nodes in each status.

A. Cycle network

A cycle network is a network that consists of a single cycle, or in other words, some number of nodes connected in a closed chain. The number of nodes in a cycle equals the number of edges, and every node has degree 2; that is, every node has exactly two edges incident with it. Taking into account the adjacency matrix of the cycle,

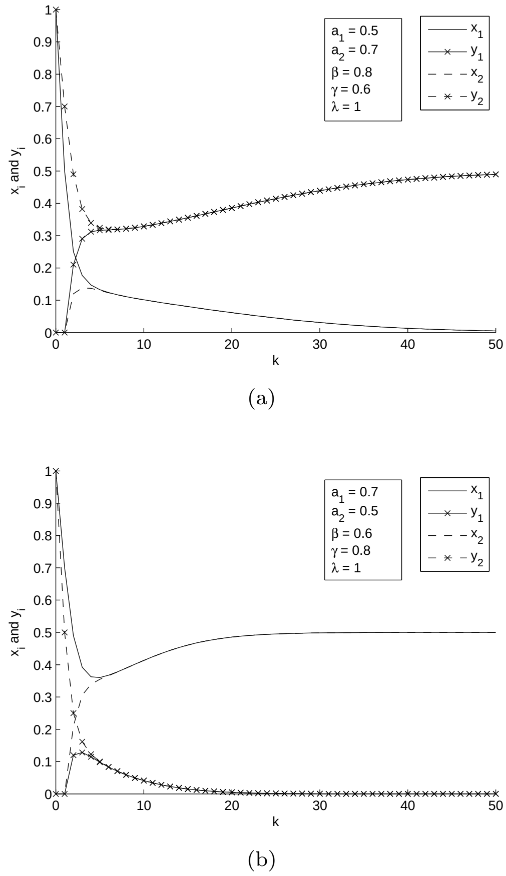


FIG. 5: The evolution of the two-consumer case (10). $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$.

(5) becomes:

$$
\begin{aligned}
x_1(k+1) &= \beta[1-x_1(k)-y_1(k)] \\
&\quad [x_2(k)+x_N(k)-\beta x_2(k)x_N(k)] + a_1x_1(k) \\
x_i(k+1) &= \beta[1-x_i(k)-y_i(k)] \\
&\quad [x_{i-1}(k)+x_{i+1}(k)-\beta x_{i-1}(k)x_{i+1}(k)]+ \\
&\quad a_1x_i(k), \\
y_1(k+1) &= \gamma[1-x_1(k)-y_1(k)] \\
&\quad [1-\beta(x_2(k)+x_N(k))+\beta^2x_2(k)x_N(k)] \\
&\quad [y_2(k)+y_N(k)-\gamma y_2(k)y_N(k)] + a_2y_1(k) \\
y_i(k+1) &= \gamma[1-x_i(k)-y_i(k)][1-\beta x_{i-1}(k)- \\
&\quad -\beta x_{i+1}(k)+\beta^2x_{i-1}(k)x_{i+1}(k)] \\
&\quad [y_{i-1}(k)+y_{i+1}(k)-\gamma y_{i-1}(k)y_{i+1}(k)]+ \\
&\quad a_2y_i(k),
\end{aligned}
\tag{11}
$$

where $i = 2, \ldots, N$, $x_{N+1} = x_1$, and $y_{N+1} = y_1$. The cycle network is a 2-regular network, so the largest eigenvalue of its adjacency matrix is $\lambda = 2$.

In the experiment presented, a cycle of 6 nodes is used. Initially, node 1 has adopted information type 1 and node 4 supports information type 2. We consider what happens when both $\frac{\beta}{1-a_1}$ and $\frac{\gamma}{1-a_2}$ are beyond the threshold value. Fig. (6) depicts a result when status 1 is the prevailing one, while Fig. (7) illustrates a case when status 2 prevails. The process of spreading first affects the two neighbors of each source, then their neighbors subsequently, and so on. As it appears, the topology of the cycle network is such that there is only one information type remaining in the graph after the dynamics of the system stabilize, even when both information types exceed the threshold value. What is more, the remaining information type has an equal probability of being supported at all the nodes. This allows for an analytic determination of the fixed point for the cycle.

When only status 1 remains probable in the network, $x_i = \tilde{x}$ and $y_i = 0$ for all nodes $i = 1, \ldots, N$. Using this in (11) we obtain three solutions for the stable value of x_i:

$$\begin{aligned}
\tilde{x}_1 &= 0 \\
\tilde{x}_2 &= \frac{\beta + 2 + \sqrt{\beta^2 - 4\beta + 4(1 + 1 - a_1)}}{2\beta} \\
\tilde{x}_3 &= \frac{\beta + 2 - \sqrt{\beta^2 - 4\beta + 4(1 + 1 - a_1)}}{2\beta}
\end{aligned}$$

Of these $\tilde{x}_1$ is stable when $\frac{\beta}{1-a_1} < \frac{1}{2}$, and $\tilde{x}_2 > 1$, which leaves $\tilde{x}_3$ for the stable value of the fixed point when $\frac{\beta}{1-a_1} > \frac{1}{2}$.

Similarly, when only status 2 remains probable in the network, we have $x_i = 0$ and $y_i = \tilde{y}$ for all nodes i. The solution of (11) is analogous:

$$\begin{aligned}
\tilde{y}_1 &= 0 \\
\tilde{y}_2 &= \frac{\gamma + 2 + \sqrt{\gamma^2 - 4\gamma + 4(1 + 1 - a_2)}}{2\gamma} \\
\tilde{y}_3 &= \frac{\gamma + 2 - \sqrt{\gamma^2 - 4\gamma + 4(1 + 1 - a_2)}}{2\gamma}
\end{aligned}$$

Again, of these three solutions $\tilde{y}_1$ is stable when $\frac{\gamma}{1-a_2} < \frac{1}{2}$, and $\tilde{y}_2 > 1$, which leaves $\tilde{y}_3$ for the stable value of the fixed point when $\frac{\gamma}{1-a_2} > \frac{1}{2}$. The result is supported by the numerical examples. Namely, the dotted lines on the figures show that the values of $\tilde{x}_3$ and $\tilde{y}_3$ indeed correspond to the values of the fixed point. Also note that these values are independent of the system size N.

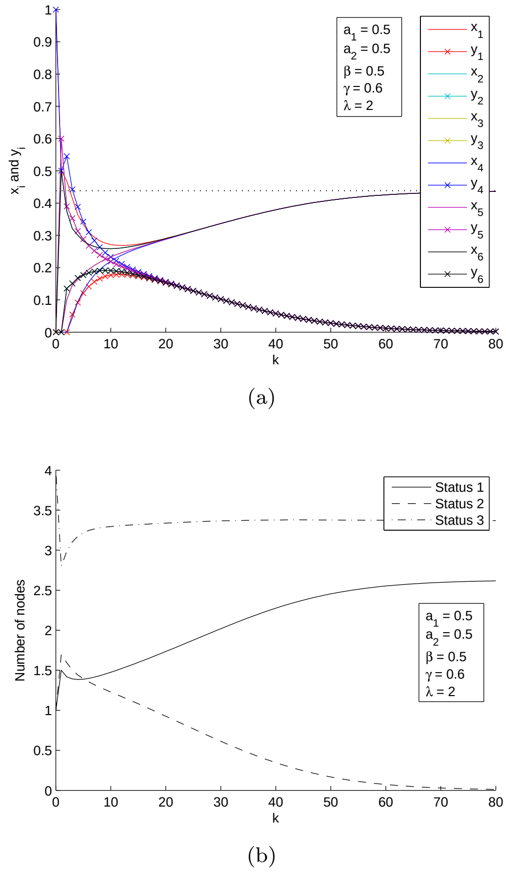


FIG. 6: The evolution of (3) for a cycle network with 6 nodes. Both information types have transmission to dissipation ratios higher than the network threshold. (a) Evolution of x_i and y_i. The dotted line is the stable value computed by $\tilde{x}_3$. (b) The average number of nodes in each status.

B. Star network

In this section results on the star topology are presented. In all cases, node 1 is the hub. For our real world examples, this network is a simplified case of a population with an "opinion leader", the hub to which all the leafs are connected. The model equations for the star become:

$$\begin{aligned}
x_1(k+1) &= [1 - x_1(k) - y_1(k)] \\
&\quad \left[1 - \prod_{j=2}^{N}(1 - \beta x_j(k))\right] + a_1 x_1(k) \\
x_i(k+1) &= [1 - x_i(k) - y_i(k)]\beta x_1(k) + \\
&\quad a_1 x_i(k), \quad i = 2, \ldots, N \\
y_1(k+1) &= [1 - x_1(k) - y_1(k)] \prod_{j=2}^{N}(1 - \beta x_j(k)) \\
&\quad \left[1 - \prod_{j=2}^{N}(1 - \gamma y_j(k))\right] + a_2 y_1(k) \\
y_i(k+1) &= [1 - x_i(k) - y_i(k)][1 - \beta x_1(k)]\gamma y_1(k) + \\
&\quad a_2 y_i(k), \quad i = 2, \ldots, N.
\end{aligned} \tag{12}$$

Fig. (8) shows the case when only one of the information types exceeds the threshold for the star for which $\lambda = \sqrt{N-1}$. Thus, only one status prevails, and all the leafs have the same value for the probability of this status. If we write $x_1 = a, x_i = b, i = 2, \ldots, N$ and $y_j = 0\ \forall j$, or $y_1 = a, y_i = b, i = 2, \ldots, N$ and $x_j = 0,\ \forall j$, depending on which the remaining status is, the fixed

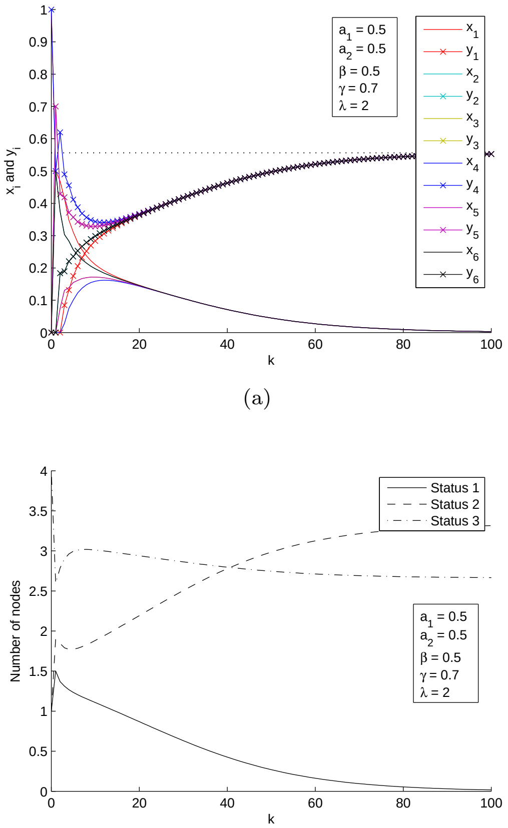


FIG. 7: The evolution of (3) for a cycle network with 6 nodes. Both β and γ are above their respective threshold parameters. (a) Evolution of x_i and y_i. The dotted line is the stable value computed by $\tilde{y}_3$. (b) The average number of nodes in each status.

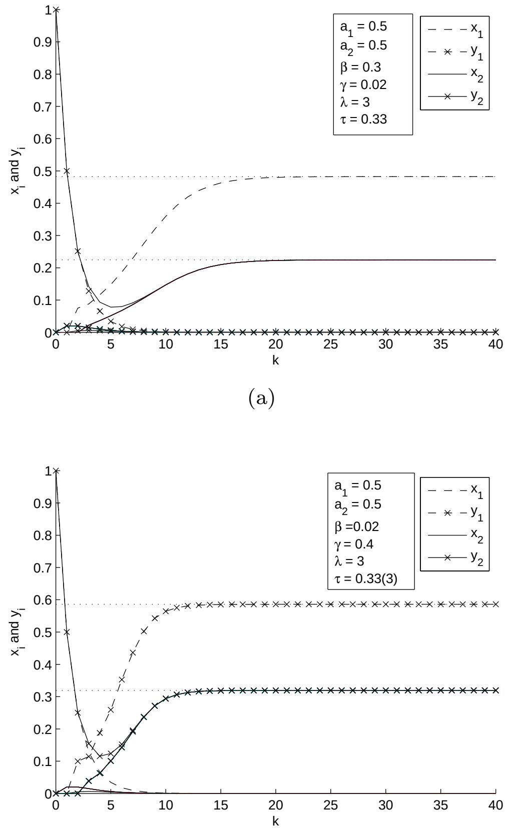


FIG. 8: The evolution of x_i (lines) and y_i (crossed lines) for a star with 10 nodes. The dotted lines are the stable values as computed by iterating (13) and (14) accordingly. (a) $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} < \frac{1}{\lambda}$. Initially the hub is in status 2, and node 2 in status 1. (b) $\frac{\beta}{1-a_1} < \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$. At $k = 0$, the hub is in status 1 and node 2 is in status 2.

point of the star, using (12), is

$$a = \frac{1-(1-\frac{\beta^2 a}{\beta a+1-a_1})^{N-1}}{1-(1-\frac{\beta^2 a}{\beta a+1-a_1})^{N-1}+1-a_1}, b = \frac{\beta a}{\beta a + 1 - a_1} \tag{13}$$

or

$$a = \frac{1-(1-\frac{\gamma^2 a}{\gamma a+1-a_2})^{N-1}}{1-(1-\frac{\gamma^2 a}{\gamma a+1-a_2})^{N-1}+1-a_2}, b = \frac{\gamma a}{\gamma a + 1 - a_2}. \tag{14}$$

Iterating these equations for the hub on $(0, 1]$ will give the correct stable values, as Fig. (8) shows.

When both information types can spread in the network, both status 1 and status 2 can appear in the stable state (Fig. (9)). Also, due to the regularity of the topology, the leaves have equal x_i and y_i values. Writing

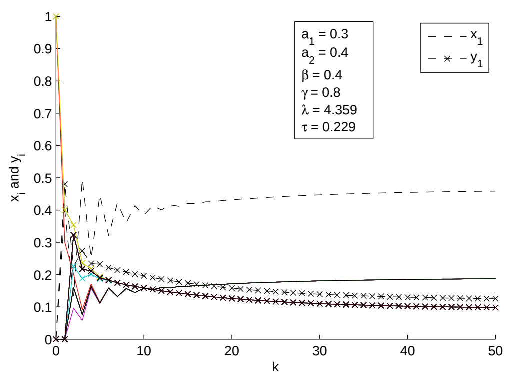


FIG. 9: Evolution of (12) for a star with 20 nodes. Initially, one leaf supports information type 1, and one leaf has adopted information type 2. Lines are for x_i and crossed lines are for y_i, $i = 1, \ldots, N$.

$(x_i, y_i) = (x_2, y_2), i = 2, \ldots, N$ in (12) we get

$$\begin{aligned} x_1 &= z_1[1 - (1 - \beta x_2)^{N-1}] + a_1 x_1, \\ x_2 &= z_2 \beta x_1 + a_1 x_2, \\ y_1 &= z_1(1 - \beta x_2)^{N-1}[1 - (1 - \gamma y_2)^{N-1}] + a_2 y_1, \\ y_2 &= z_2(1 - \beta x_1)\gamma y_1 + a_2 y_2, \end{aligned}$$

where $z_1 = 1 - x_1 - y_1$ and $z_2 = 1 - x_2 - y_2$. From the last four equations we can see the behavior of the model in the limit of large N. In fact $(1 - \beta x_2)^{N-1} \approx 0$ when N is large, and it follows that $y_i \approx 0$, $\forall i$, while

$$x_1 = \frac{1}{2 - a_1}, x_2 = \frac{\beta}{\beta + (1 - a_1)(2 - a_1)}. \quad (15)$$

This is shown on Fig. (10), for a star with 1000 nodes. In words, in such a situation it is very difficult for information type 2 to pervade the network. Once the hub hears the word from someone in status 1, it spreads it to all the other nodes in the network, and they will always consider this message first.

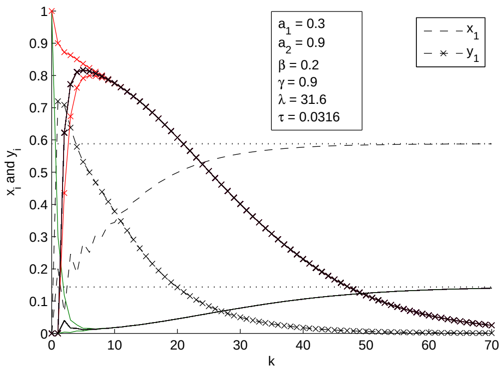


FIG. 10: The evolution of (12) for a 1000-node star. Initially, one of the leafs is in status 1, and one is in status 2. Lines are for x_i and crossed lines are for y_i. The dotted lines are approximations for the stable values of the hub and leaves, as given by (15).

C. Fully connected network

A complete (or fully connected) network is a simple network in which every pair of distinct nodes is connected by an edge. The fully connected network is an example of an environment with homogeneous mixing where every individual has equal contact with the others. The model equations for this topology become:

$$\begin{aligned} x_i(k+1) &= [1 - x_i(k) - y_i(k)] \\ &\quad \left[1 - \prod_{j=1, j\neq i}^{N}(1 - \beta x_j(k))\right] + a_1 x_i(k) \\ y_i(k+1) &= [1 - x_i(k) - y_i(k)] \prod_{j=1, j\neq i}^{N}(1 - \beta x_j(k)) \\ &\quad \left[1 - \prod_{j=1, j\neq i}^{N}(1 - \gamma y_j(k))\right] + a_2 y_i(k) \end{aligned} \quad (16)$$

We present several results. Fig. (11) shows cases when only one of the information types has a transmission to dissipation ratio greater than τ. Accordingly, only one type of message is able to spread through the network. The stable values x_i or y_i are the same for all the nodes. Using this in (16) we obtain:

$$\tilde{x} = \frac{1 - (1 - \beta\tilde{x})^{N-1}}{1 - (1 - \beta\tilde{x})^{N-1} + 1 - a_1} \quad (17)$$

or

$$\tilde{y} = \frac{1 - (1 - \gamma\tilde{y})^{N-1}}{1 - (1 - \gamma\tilde{y})^{N-1} + 1 - a_2} \quad (18)$$

for the value of the stable state in Fig. (11a) and Fig. (11b). Iterating each of these equations on $(0, 1]$ instead of (16) will also give the stable state solution. When N is large enough, the solutions for the one-status-spreading case become independent of the message transmission rates, and are well approximated by:

$$\begin{aligned} \tilde{x} &= \tfrac{1}{2 - a_1} \\ \tilde{y} &= \tfrac{1}{2 - a_2} \end{aligned} \quad (19)$$

When both $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$, both message types are able to spread through the network. For the parameter values on Fig. (12), each node has the possibility of being in each of the three statuses, and on average most nodes will be in status 2. Due to the regularity of the topology, all the nodes have the same stable values for all the probabilities. Using this fact, from (16) one obtains

$$\begin{aligned} \tilde{x} &= (1 - \tilde{x} - \tilde{y}) \left[1 - (1 - \beta\tilde{x})^{N-1}\right] + a_1 \tilde{x} \\ \tilde{y} &= (1 - \tilde{x} - \tilde{y})(1 - \beta\tilde{x})^{N-1} \left[1 - (1 - \gamma\tilde{y})^{N-1}\right] + a_2 \tilde{y} \end{aligned} \quad (20)$$

for the value of the fixed point. An interesting observation is that when N is large enough, the fixed point becomes

$$\begin{aligned} \tilde{x} &= \tfrac{1}{2 - a_1}, \\ \tilde{y} &= 0. \end{aligned} \quad (21)$$

An illustration of this is in Fig. (13), for a fully connected network of 100 nodes and one node in each status initially. Although messages of type 2 pervade the network at the beginning due to high a_2 and γ, status 2 is not sustained in the network. In the words of a real world example, this means that in a large enough market in which all individuals are in contact with each other, w-o-m about a new product from an unknown company will have no chances against the brand-name product.

A note on the results for large stars and fully connected graphs is in order. In the limit of large N, the hub and the fully connected nodes adopt either status 1 or an undecided status. The proportion of time spent in these statuses is $\frac{1}{1+1-a_1}$ and $\frac{1-a_1}{1+1-a_1}$, respectively. The numerators in the expressions signify the transmission rate and dissipation rate of information at the particular

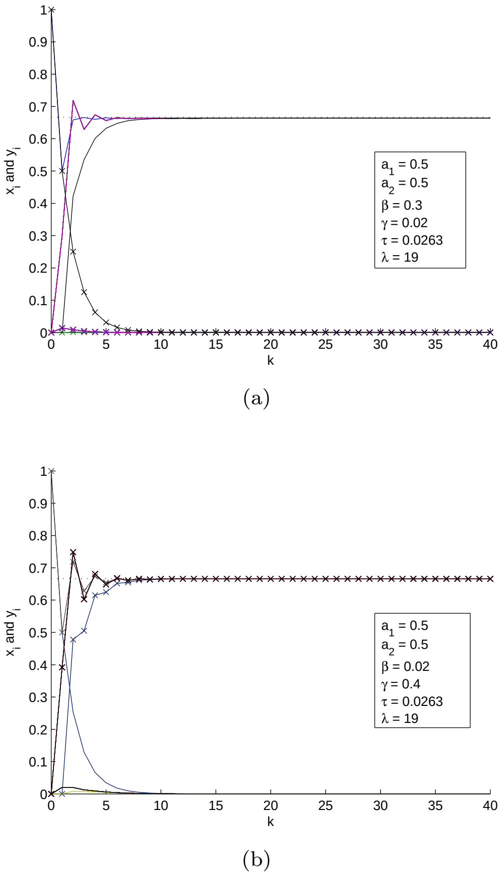


FIG. 11: The evolution of x_i (solid lines) and y_i (crossed lines) for a 20-node fully connected network. The dotted lines are the approximated stable values (19). (a) Only information type 1 has parameters above the network threshold. (b) Only information type 2 has parameters above the network threshold.

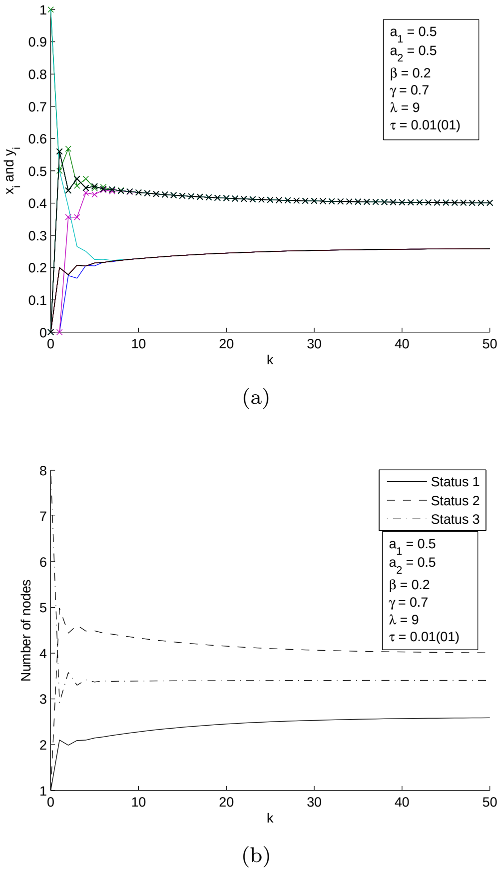


FIG. 12: The evolution of (16) for a fully connected network with 10 nodes. Both message types can pervade the network. (a) Evolution of x_i, depicted by solid lines and y_i, depicted by crossed lines. (b) The average number of nodes in each status.

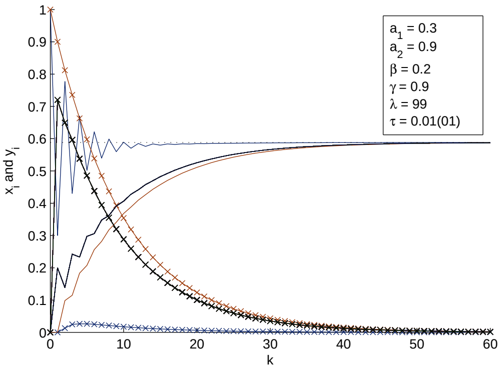
node. As can be seen, when a node has a high degree, the rate of receiving a message from its many neighbors is 1, regardless of the actual message transmission parameter. In fact, one can argue that this happens in an arbitrary topology as well. For a node i with high enough degree, the product

$$\prod_{j=1}^{N}(1-\beta a_{ij}x_j(k))$$

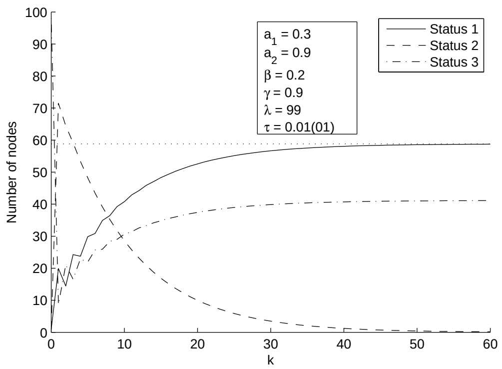
in (4) tends to 0, i.e. $f_i \approx 1$ in the model equations, yielding the steady state values as observed in the large star and fully connected graph. The degree value above which this behavior is observed depends on the values of the model parameters, as well as on the network topology. For example, consider Fig. (10) and Fig. (13). For the same model parameters, the fully connected graph requires much less nodes than the star in order to observe the stable values $\tilde{x} = \frac{1}{2-a_1}$ and $\tilde{y} = 0$. The fully connected topology allows the nodes to reinforce their decision more easily.

IV. BEHAVIOR OF THE MODEL ON COMPLEX NETWORK TOPOLOGIES

We now investigate the model behavior for complex network topologies. Of interest to us is the case when $\frac{\beta}{1-a_1} > \frac{1}{\lambda}$ and $\frac{\gamma}{1-a_2} > \frac{1}{\lambda}$, since theory and simulations both indicate that this is the only case when both information types are able to spread in the networks. All experiments start with one node in status 1 and one in status 2, which can change their status as time progresses. Moreover, in almost all experiments information type 2 has relatively high parameter values in comparison to type 1, so as to observe its spread in the networks. Where applicable, we relate the results with those on simple networks.



(a)



(b)

FIG. 13: The model evolution for a 100-node fully connected network. (a) Evolution of x_i, depicted by solid lines and y_i, depicted by crossed lines. Dotted line is the approximating stable value for x_i given by (21). (b) The average number of nodes in each status. Dotted line is the stable number of nodes in status one, as predicted by $N_1 = N\tilde{x} = 100/(2 - a_1)$.

A. Erdos-Renyi random networks

In this section we observe the model behavior on Erdos-Renyi (ER) networks. The model proposed by Erdos and Renyi describes random graphs with N nodes in which every link exists with probability p. The degree distribution of these networks is Poisson, hence the homogeneous structure in the sense that all nodes have degree close to the average degree. Also, in this model there is a critical probability value $p_c = \frac{1}{N}$ under which the resulting network consists of small disconnected components, and above which there is a giant component in the network containing $O(N)$ nodes. All the networks used in the simulations are generated with $p > p_c$, and the sources of information are randomly placed in the giant component. Fig. (14) shows the steady state behavior of the model for ER networks with 1000 nodes for different values of p. Increasing p, the nodes have an increasingly higher degree, which, as can be seen, serves to promote information of type 1 at the expense of information of type 2, as could be expected from our previous discussion. The number of nodes in status 1 is accurately predicted by the approximation $N_1 = N\tilde{x}, \tilde{x} = \frac{1}{2-a_1}$. In the context of our real-world example, information about the product from a lesser known brand has chances to penetrate a random social network when the average degree of the nodes is low enough to allow it.

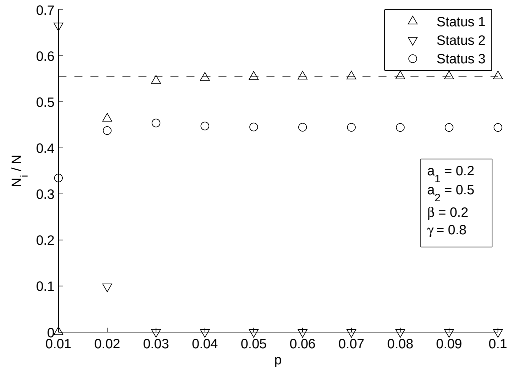


FIG. 14: The steady state behavior of the model for ER networks with $N = 1000$ for different values of p, depicting the fraction of nodes in each status. For every value of p the results are the averages of 100 network realizations, each having a giant component with size of at least $0.9N$. The dashed line is the approximation of the number of nodes adopting information type 1 given by $N_1 = N\tilde{x}, \tilde{x} = \frac{1}{2-a_1}$.

B. Small-world graphs

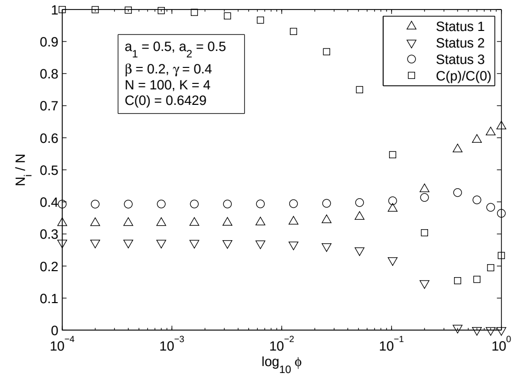
In this section we investigate how the model behaves on small-world graphs. It has been suggested that many real-world networks, and social networks among them have small-world characteristics. We use the Watts-Strogatz model as defined in [30] for generating the networks. ϕ denotes the rewiring parameter in the model, instead of β, to avoid confusion with the model parameter of this article. The algorithm uses a starting ring lattice to construct a small-world network. In a ring lattice each node has $2K$ neighbors, K in the clockwise and K in the anti-clockwise direction. Each edge is rewired with probability ϕ, not allowing self-loops or multiple edges between nodes.

Fig. (15) depicts the steady state behavior of the model when the rewiring parameter ϕ is varied. The results are shown for networks of three different sizes for which $C(0)$ - the clustering coefficient of the initial ring lattice is the same. The fraction of nodes in each status is given, as well as the clustering coefficient of each network, normalized by $C(0)$. Evidently, the results are the same for all three network sizes, due to the same level of clustering in the networks. Furthermore, for the particular values of the model parameters, status 2 is also present in the small-world networks. The cliquishness of the environment enables information type 2 to occupy some of the nodes. However, as the networks become increasingly random, status 2 disappears. The lack of significant clustering in the random networks seems to undermine the spreading of the second type of information, while the preferred information type 1 easily pervades the network.

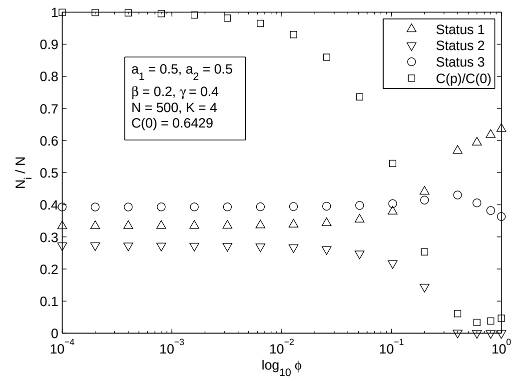
Furthermore, if the level of clustering in the networks is higher, i.e. the cliquish neighborhoods are larger, as in Fig. (16), then status 1 is the only remaining status, even if the parameters of information type 2 are relatively higher than those of type 1. The high clustering of the networks acts to strengthen the influence of status 1, a result reminiscent of that in a fully-connected graph. We further illustrate this by providing the time behavior of the model for a 1000-node ring lattice with $K = 10$ (Fig. (17)), and a small-world network generated from it with $\phi = 0.01$ (Fig. (18)), using the same parameters as in the previous experiment.

The ring lattice is a highly clustered graph. Every node has a cliquish neighborhood, and specifically, in this example each 11 consecutive nodes on the ring are fully connected. Fig. (17a) shows the evolution of the model (5) for nodes 1 to 11. Since in this case node 51 is in status 2 initially, in the beginning the group is influenced by the messages of type 2. The ring lattice has a long average path length. With the source of information type 1 being node 501, it takes a longer time for the type 1 messages to traverse the network and reach the observed group. Once they do, however, information of type 1 suppresses the influence of type 2. Further, the values of x_i and y_i at the clique of 11 nodes are well approximated by the equations (19) for a fully connected graph. Globally, Fig. (17b) shows that both information types progressively affect nodes, with status 2 being slightly more successful in the beginning, due to its higher message transmission and remembrance parameters. In the long run, however, information type 1 is greatly promoted due to the cliquishness of the environment, becoming the only information type in the network.

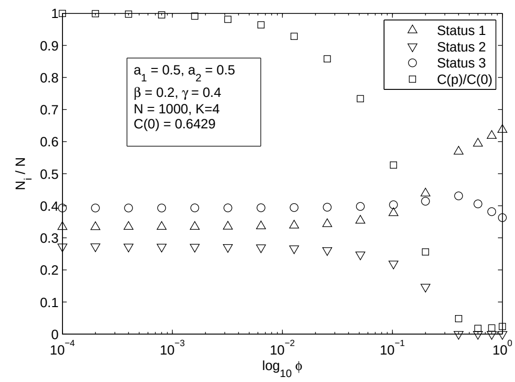
We can now compare the behavior of the model on small-world networks with that on the ring lattice. Fig. (18) shows a small-world network generated from the ring lattice in the previous example, with $\phi = 0.01$. The initial conditions are the same as previously. The most obvious difference with the regular case is the speed of the dynamics. The process converges much more quickly on a small-world network due to the short-cut connections which greatly reduce the average distance between nodes. As a result, the spreading of the type 2 messages is greatly facilitated, and many nodes adopt information type 2 in the beginning. However, there is a rapid spread in type 1 messages as well, and combined with the high clustering of this network, status 1 being preferable easily suppresses status 2. The end result is again reminiscent of that in fully connected graphs, with the stable



(a) $N = 100$



(b) $N = 500$



(c) $N = 1000$

FIG. 15: The steady state behavior of the model for different values of ϕ. The fraction of nodes in each status is depicted, as well as the normalized clustering coefficient for the networks. (a), (b) Results obtained by averaging over 50 network realizations for each ϕ, run for $k = 400$ time units. (c) Results obtained by averaging over 25 network realizations run for $k = 700$ time units.

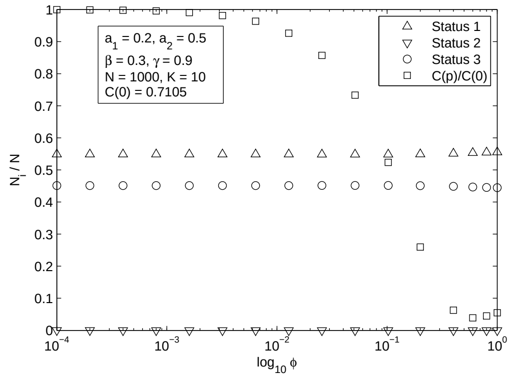
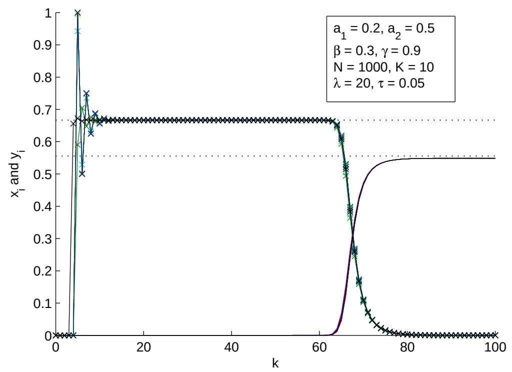
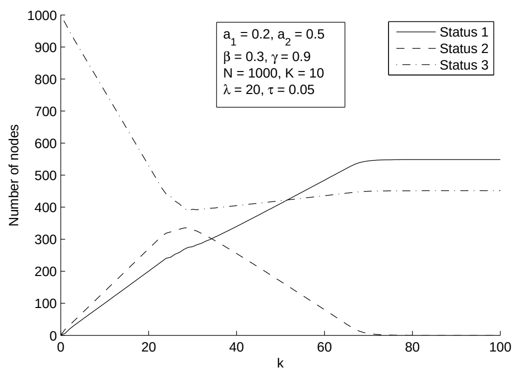


FIG. 16: The steady state behavior of the model for networks generated with the Watts-Strogatz algorithm, using a starting ring lattice with 1000 nodes and $K = 10$.



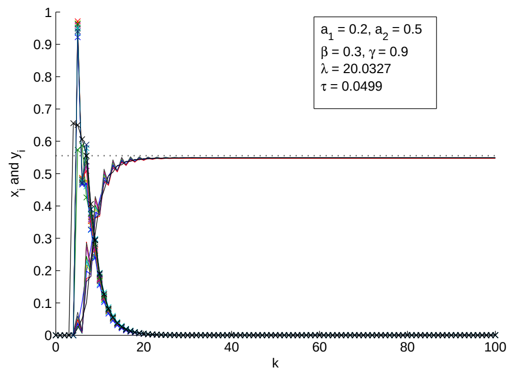
(a)



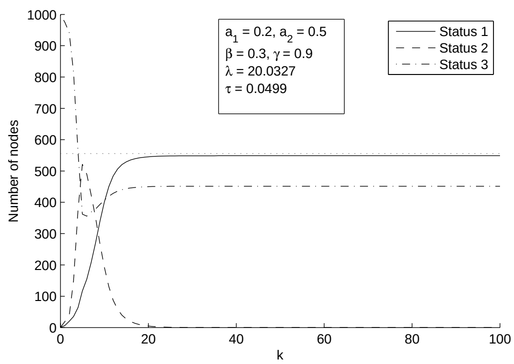
(b)

FIG. 17: Behavior of the information spreading model (5) on a 1000-node ring lattice with $K = 10$. Initially, node 51 is in status 2 and node 501 is in status 1. (a) The evolution of x_i and y_i for $i = 1, 2, \ldots, 11$. Solid lines stand for x_i and crossed lines stand for y_i. Dotted lines are the approximating values of x_i and y_i for a fully connected graph as given with (19). (b) The average number of nodes in each status.

x_i value of the observed nodes 1-11 being well approximated by $\tilde{x} = \frac{1}{2-a_1}$. Also, the total number of nodes in status 1 when the model stabilizes is well approximated by $N_1 = N\tilde{x}$.



(a)



(b)

FIG. 18: Behavior of the information spreading model (5) on a small world network with 1000 nodes generated from a ring lattice with $K = 10$ using $\phi = 0.01$. Initially, node 51 is in status 2 and node 501 is in status 1. (a) The evolution of x_i and y_i for $i = 1, 2, \ldots, 11$. Solid lines stand for x_i and crossed lines stand for y_i. The dashed line is the stable value for x_i approximated by the expression in (19) for a fully connected graph. (b) The average number of nodes in each status, obtained by summing the probabilities (3) for all nodes. Dotted line is the approximation $N_1 = N\tilde{x}$.

In conclusion, large neighborhoods in a small-world network which are clustered highly enough behave as fully connected graphs, strengthening the influence of the preferred information type 1, and disabling the spread of information type 2. Messages of type 1 are also easily spread through the long-range connections, quickly affecting the whole network. In the context of our real-world examples, word about a new product in the presence of an already established one could only be successfully spread if a small-world social network does not have large highly connected groups of individuals.

C. Scale-free graphs

In this section results of the model behavior on scale-free graphs are given. The original BA algorithm as given in [28] is used to construct scale-free networks. One starts from a seed of m_0 connected nodes and adds a new node with $m \leq m_0$ links at each step according to the preferential attachment rule.

Fig. (19) shows the average number of nodes in each status as a fraction of N, as the parameter m of the BA algorithm is varied. The seed for the BA algorithm is a 20-node fully connected graph. For each value of m, the largest hub in the network has a degree high enough to act as a star, having $\tilde{x} = \frac{1}{2-a_1}$ and $\tilde{y} = 0$. However, the total number of nodes in status 1 is different for different values of m in this particular setting. For $m = 1$ and $m = 2$ status 2 prevails, suggesting that the largest hub of the network does not have a far-reaching influence. Increasing m brings an increasing number of nodes in status 1, mostly at the expense of those in status 2, eventually expelling information of type 2 from the network for $m \geq 6$. As m is increased the algorithm creates a network with more hubs and higher degree nodes in general, the combination of which seems to promote the preferred information type 1 heavily. One can imagine as m increases, the number of nodes in status 1 to approach the dashed line.

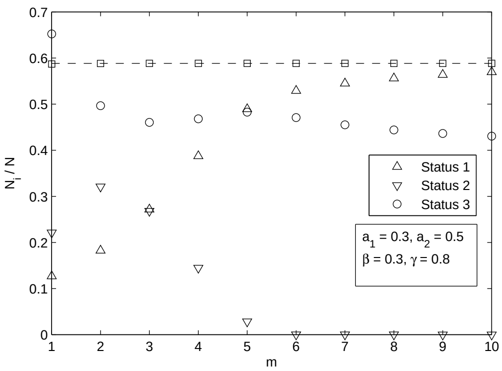


FIG. 19: The average number of nodes in each status as a fraction of N, as m is varied. For each value of m, results are averaged over 100 network realizations, with the stable values taken at $k = 100$. All networks are generated from a 20-node fully connected seed and have $N = 1000$. The stable x-value for the largest hub in the networks (squares) correspond to the approximation $\tilde{x}$ for the star in (15) (dashed line).

V. CONCLUSIONS AND DISCUSSION

In this paper we make an attempt to study how two different kinds of information, one of which is always considered first, propagate in a network. The model presented describes the interactions among nodes, and general results for the interplay of the two information types on different topologies are given. The key points of this paper are as follows.

- We suggest a novel model of information (rumor) spreading over networks, in which each node can be in one of three possible states: 1, 2 and 3 corresponding to class-one spreader, class-two spreader and ignorant, respectively. The model is a natural generalization of the well known epidemic SIS model, and reduces to the SIS model when some of the model parameters are zero. We confirm the existence of an intrinsic network threshold $\frac{1}{\lambda}$ for the spreading process to occur, as is already suggested by previous studies, where λ is the largest eigenvalue of the network's adjacency matrix A. The model has a unique stable fixed point, which implies irrelevance of the choice of initial information spreaders.

- We find that the preferred information type 1 is heavily promoted when the degree of nodes is high enough and/or when the network contains large clustered groups of nodes, expelling information type 2. This is reflected in the model behavior on simple network topologies as well as on complex networks. Specifically, increasing the average degree in ER graphs yields the same results, as does increasing the cliquish neighborhoods in small-world networks generated by the Watts-Strogatz model. In both cases the number of nodes adopting information type 1 is well approximated by $N\tilde{x}, \tilde{x} = \frac{1}{2-a_1}$. The behavior of the model is similar in BA networks as m is increased.

- From the standpoint of the source of information type 2, having to compete with a more reputable source is extremely challenging in many cases. Nevertheless, simulations show that it is possible for information type 2 to occupy a nonzero fraction of the nodes in many cases as well. Specifically, in the Watts-Strogatz small-world model some level of clustering facilitates its adoption, while increasing randomness undermines it. For ER networks, a low average degree allows the coexistence of the two types of information. BA networks generated with a small m are also a pervading substrate for information type 2.

Future research directions are numerous. Conditions in which status 2 has a "win situation" over status 1 remain to be determined. The stationary distribution of the Markov process for the network might be a promising tool for this matter. Non-ergodic versions of the model, i.e. the ones when condition (7) doesn't hold, are also worth investigating. The problem of choosing the initial set of information spreaders which maximize the number of nodes in status 1 and 2 is natural in that setting. Finally, how well the model approximates reality is a key question to its usefulness. Hence, comparison with available real world data is in order.

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