LIUC

MANMADE

DIAGNOSING VULNERABILITY, EMERGENT PHENOMENA, and VOLATILITY in MANMADE NETWORKS

European Commission
Joint Research Centre

From complex networks to time series and *viceversa*

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Outline

1. Introduction
2. Methods and Approach
3. Preliminary results

   - From Time Series to Networks: Lorenz (Chaotic) versus Scale Free Networks

   - From Networks to Time Series: Gaussian and long tail probability distribution functions

NEST Pathfinder

Man made

Introduction

- Objectives:
  - Development of a general framework to transform complex networks into non-linear time series
  - Explore new research avenues that this dual vision of complex phenomena can provide us

Introduction

- Existing Approaches:

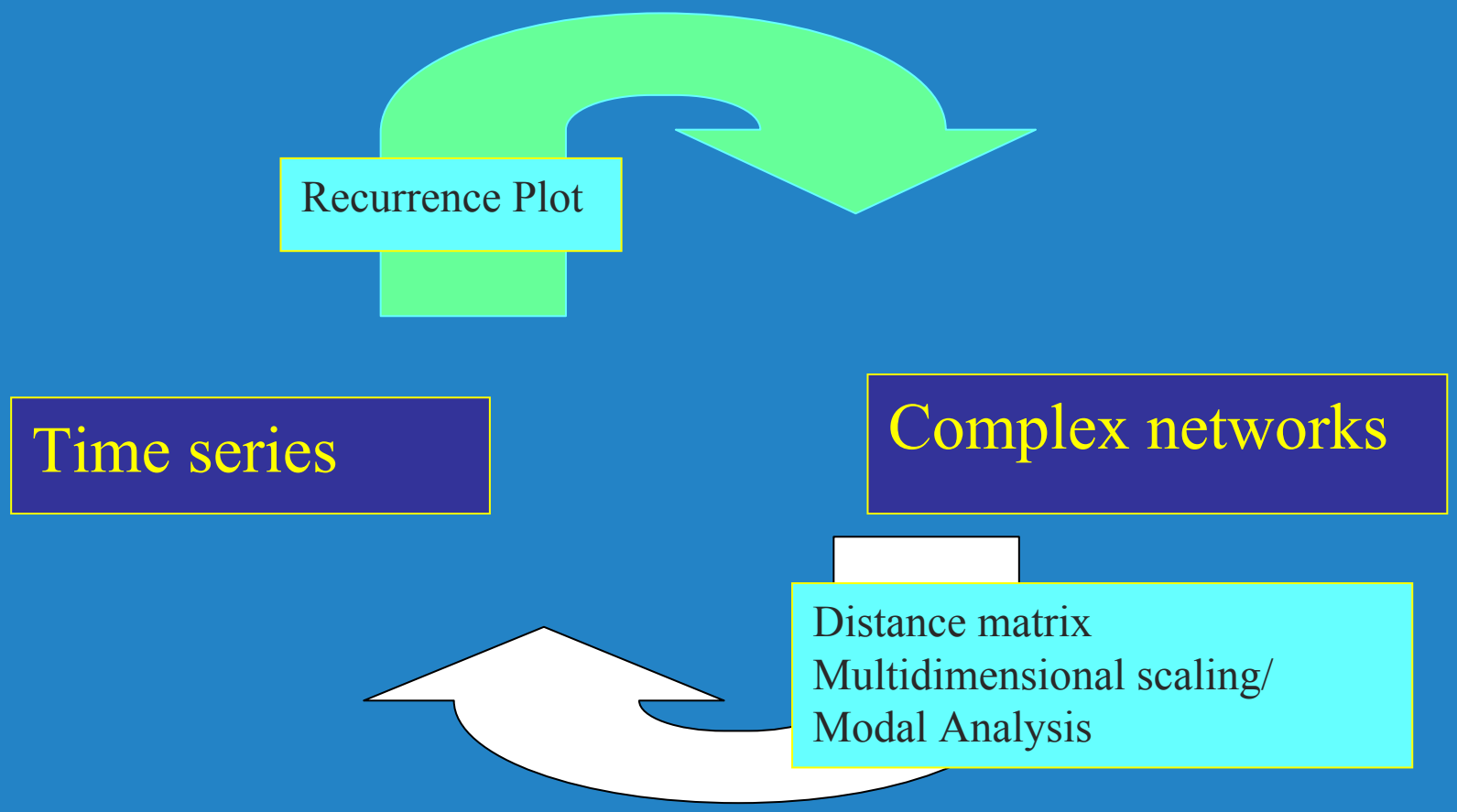
From TS to NET

- Hirata and Aihara, 2008. Riproduction of distance matrices and original time series from recurrence plots and their applications. Phys. J. Special Topics 164, 13-22
- Lacasa et al. 2008. From time series to complex networks: the visibility graph, PNAS 103 , 4972-4975
- Zhang and Small. 2006. Complex Network from Pseudoperiodic Time Series: Topology versus Dynamics Phys. Rev. Lett. 96, 238701
- Xu, Zhang and Small, 2009. Superfamily phenomena and motifs of networks induced from time series. PNAS 105, 19601-19605

From NET to TS

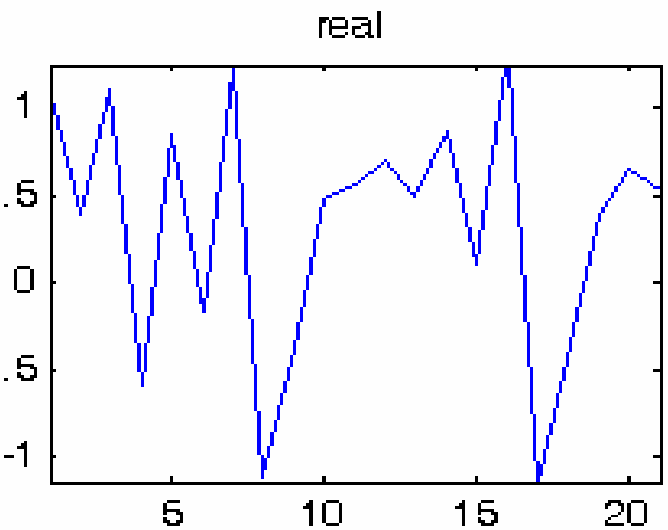
- Hirata and Aihara. 2008. Riproduction of distance matrices and original time series from recurrence plots and their applications. Phys. J. Special Topics 164, 13-22

Methods and Approach

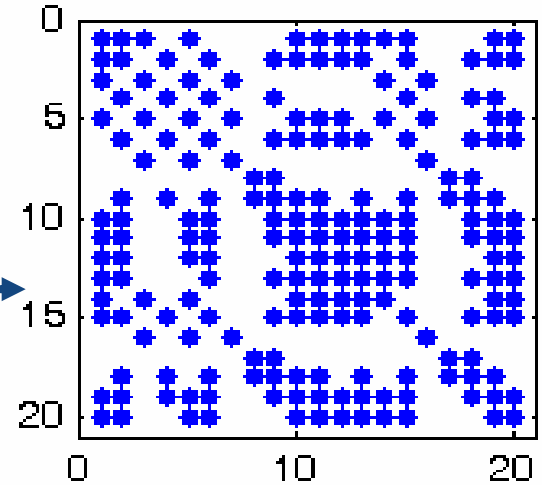


Method 1: Multidimensional scaling (Hirata and Aihara, 2008)
Method 2: Modal analysis (Gutiérrez and Zaldívar, 2000)

Methods and Approach



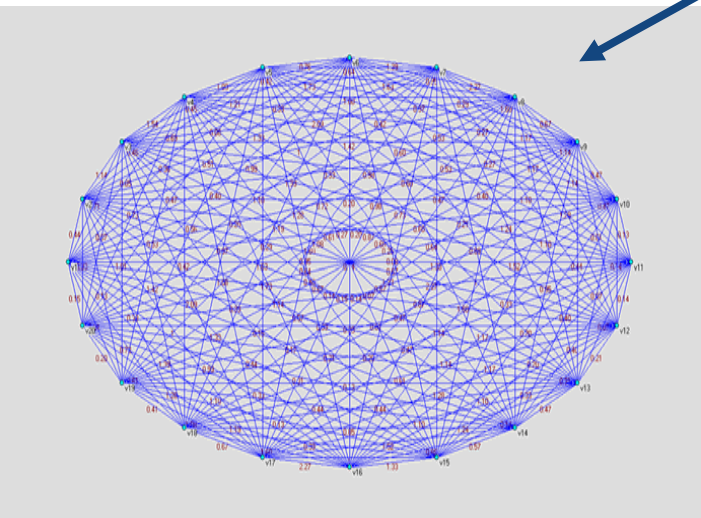
Recurrence Plot



1

Weighted Graph/
Min. distance matrix

2



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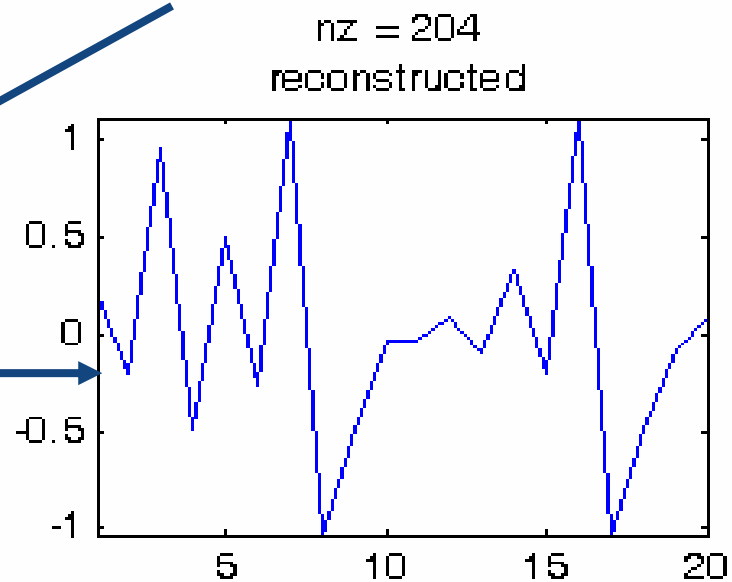


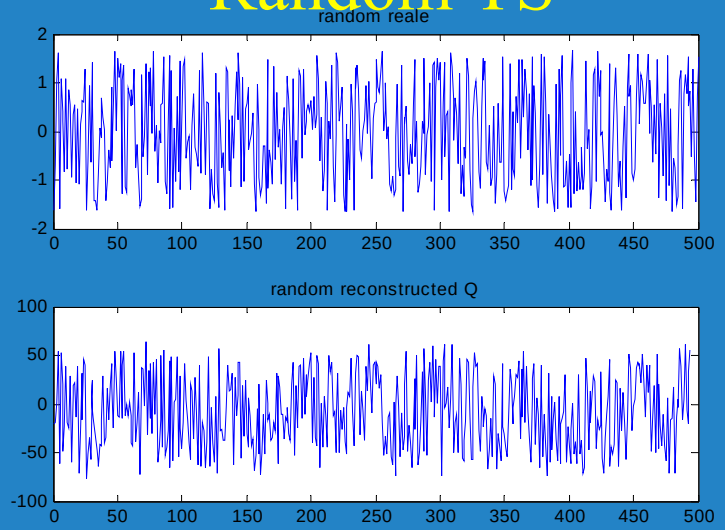
$$1/\ G_i = \{ j : c(z_i, z_j) = 1 \}$$

$$2/\ w(i,j) = 1 - \frac{|G_i \cap G_j|}{|G_i \cup G_j|}$$

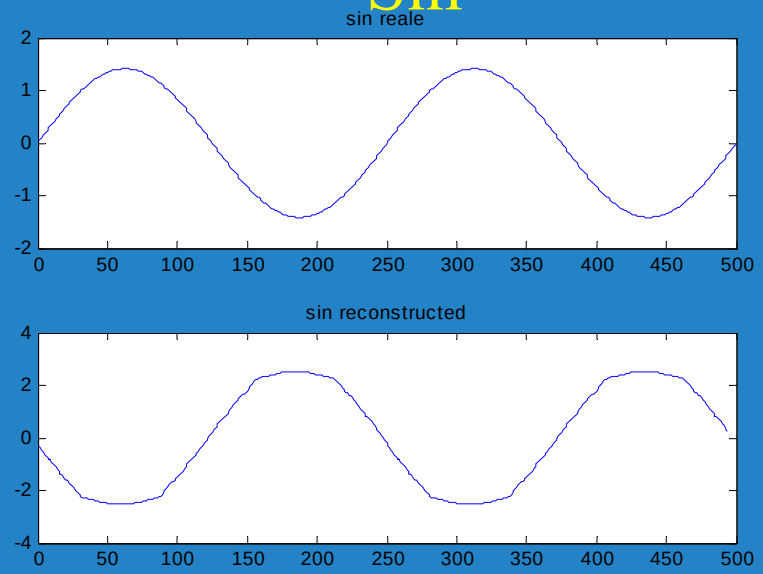
3/ - Method 1: Multidimensional Scaling
- Method 2: Modal Analysis

Methods and Approach: Examples

Random TS

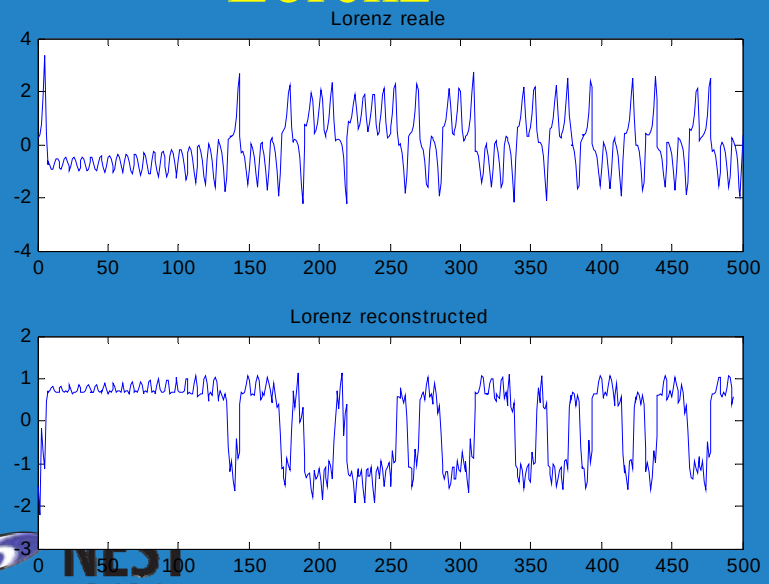


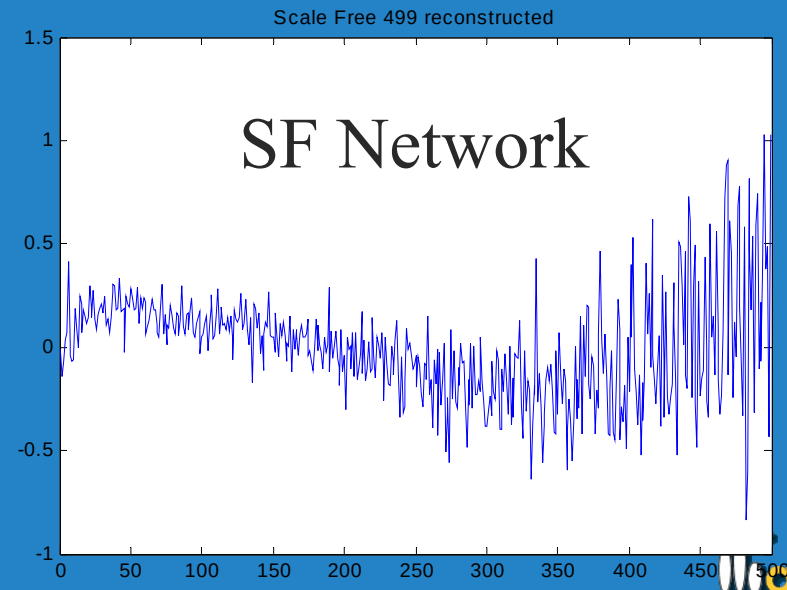
Sin



Method 1

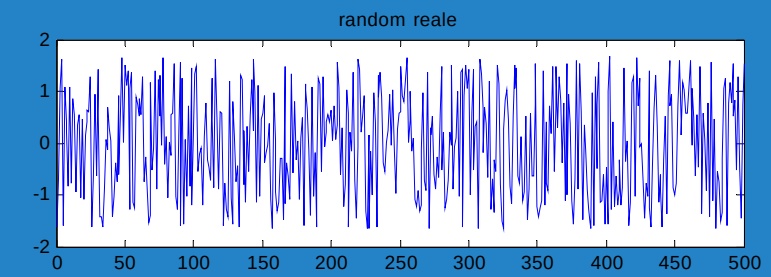
Lorenz

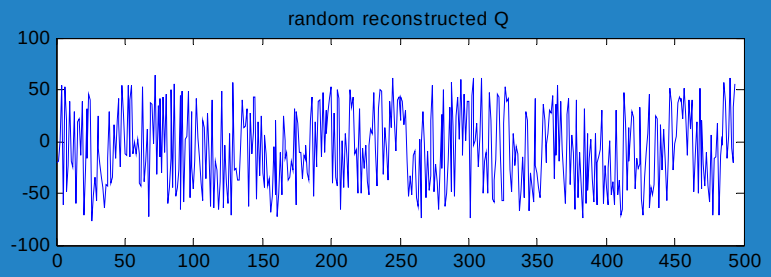




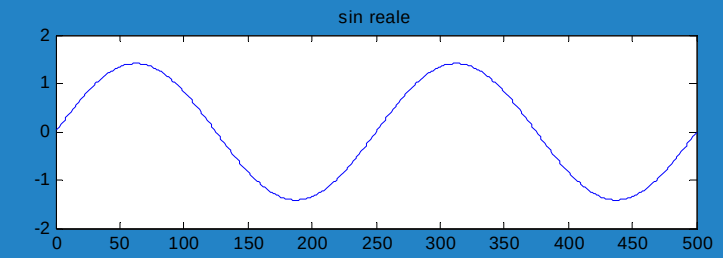
Methods and Approach: Examples

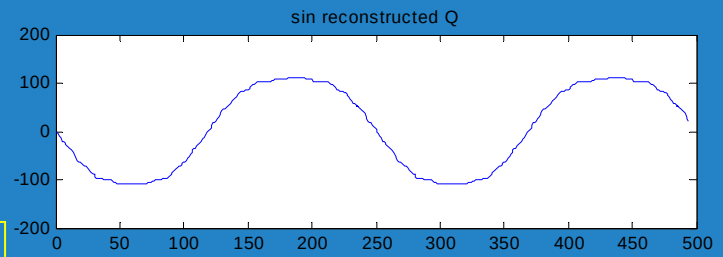
Random TS





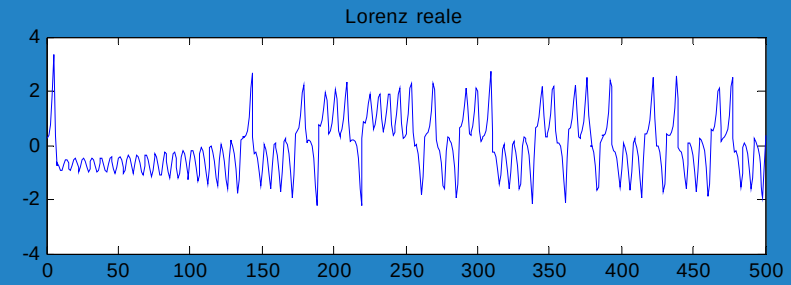
Sin TS

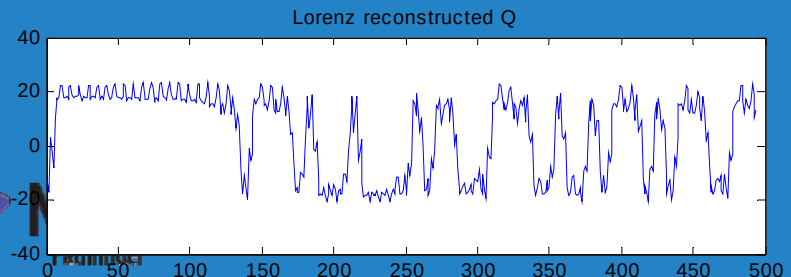


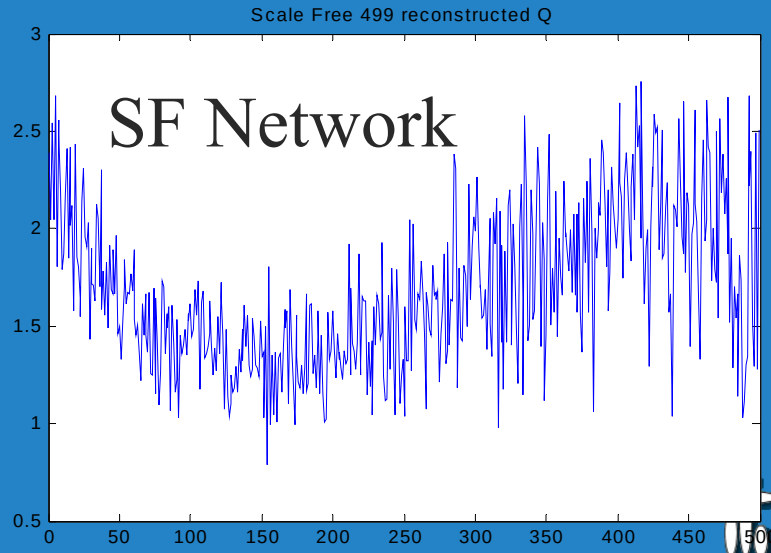


Method 2

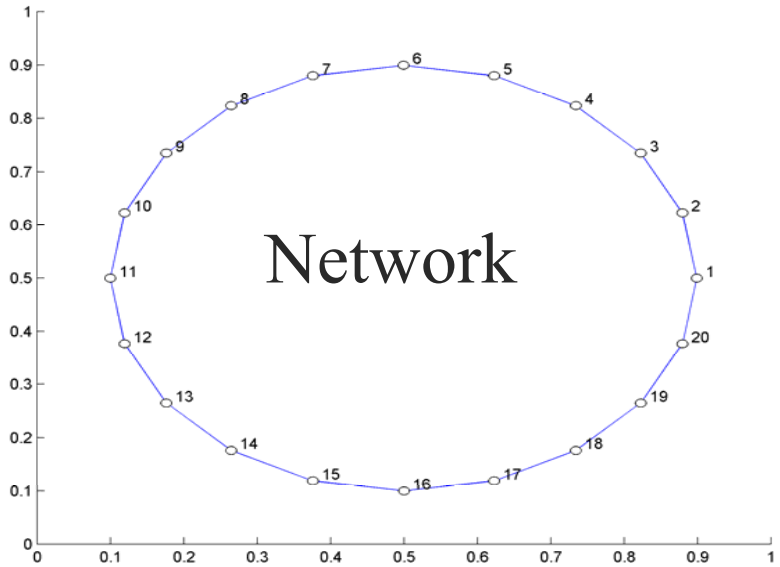
Lorenz TS

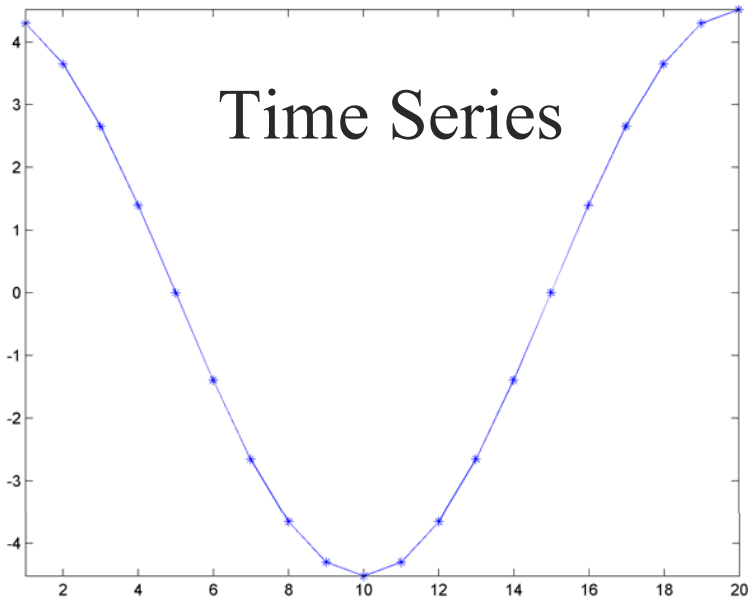


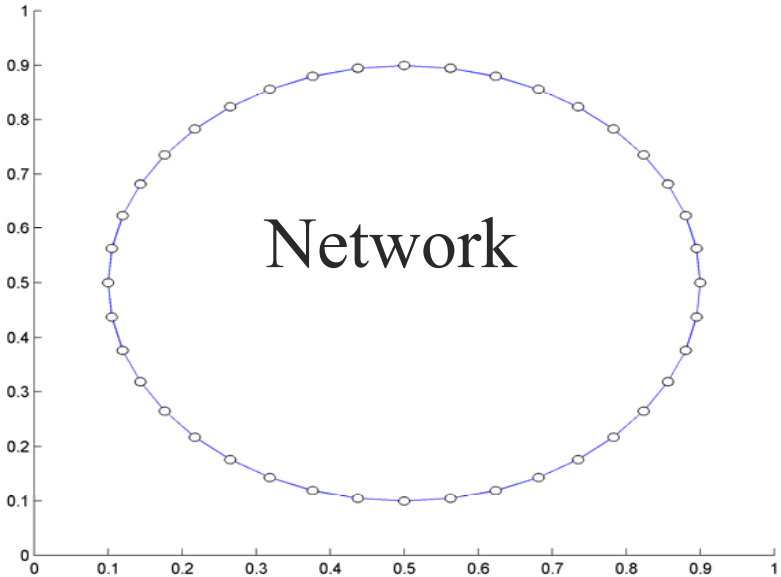


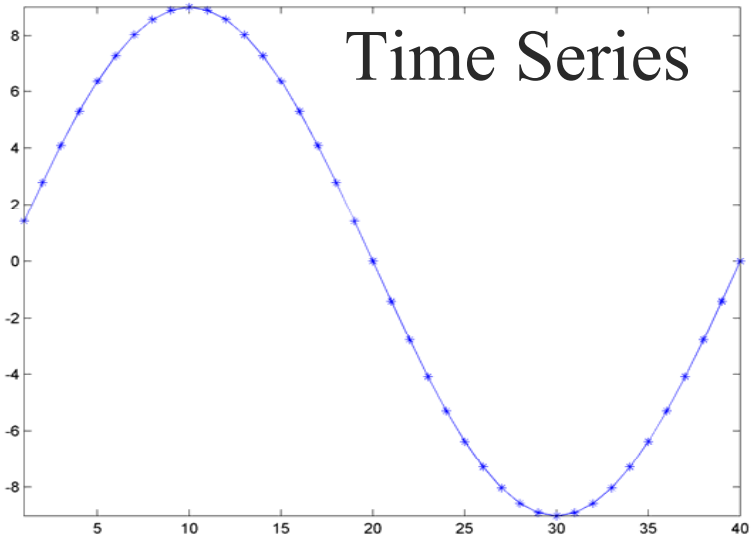


Methods and Approach: Examples







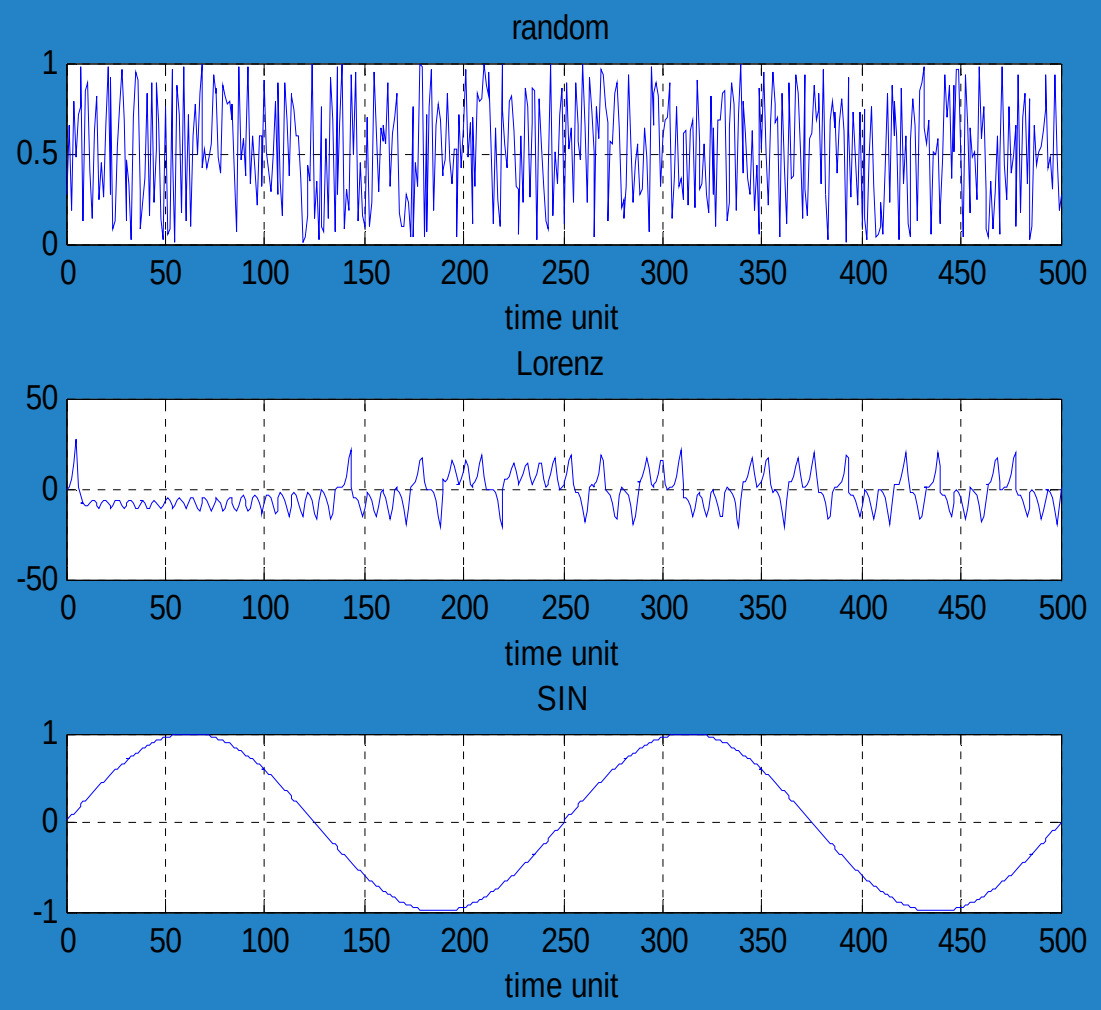


Preliminary results

Part A: From time series to networks

Lorenz versus Scale Free networks

Preliminary results



Uniform distribution
on the unit interval [0, 1]

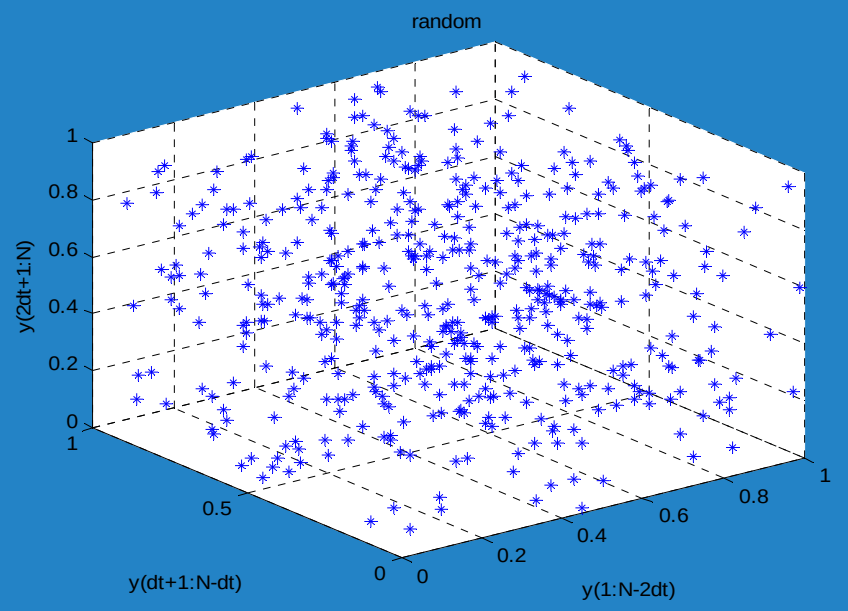
y=rand(1,500)

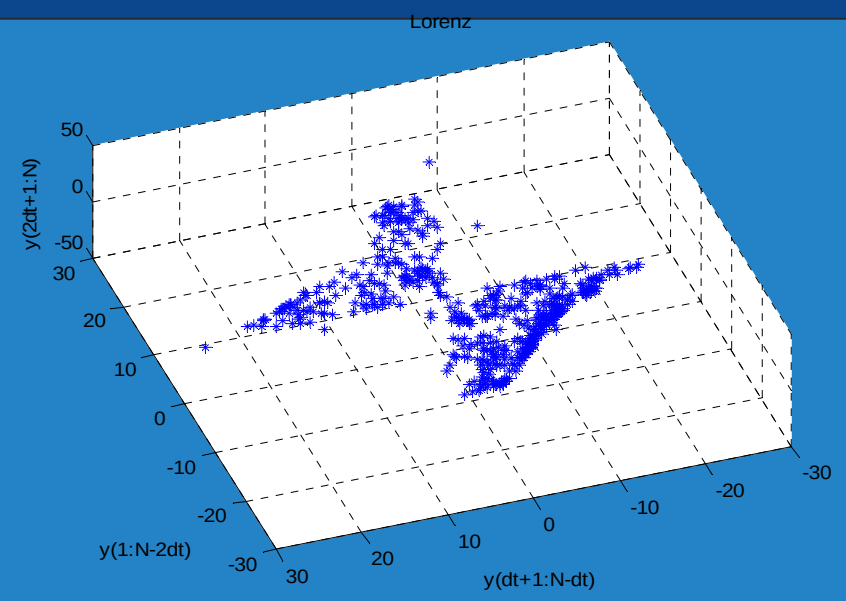
$$\begin{cases} \dfrac{dx(t)}{dt} = \sigma\big(y(t) - x(t)\big) \\ \dfrac{dy(t)}{dt} = -y(t) + (r - z(t)) \cdot x(t) \\ \dfrac{dz(t)}{dt} = -b \cdot z(t) + x(t) \cdot y(t) \end{cases}$$

$r=28,\ b=8/3 \quad \sigma = 10$

t=[0:5 *pi/500:5*pi]; y=sin(t);

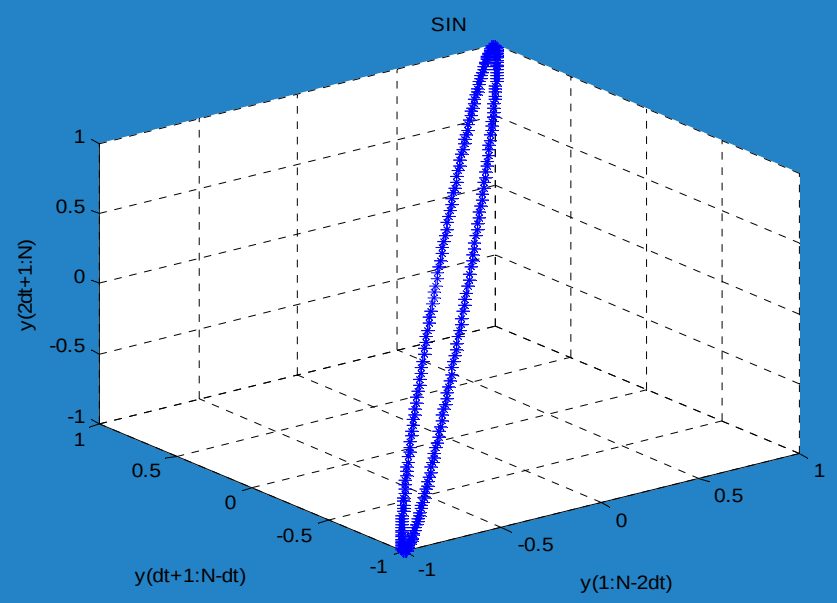
Preliminary results





Reconstruction with:

$\Delta t=3$; $d_E=3$; $N=500$



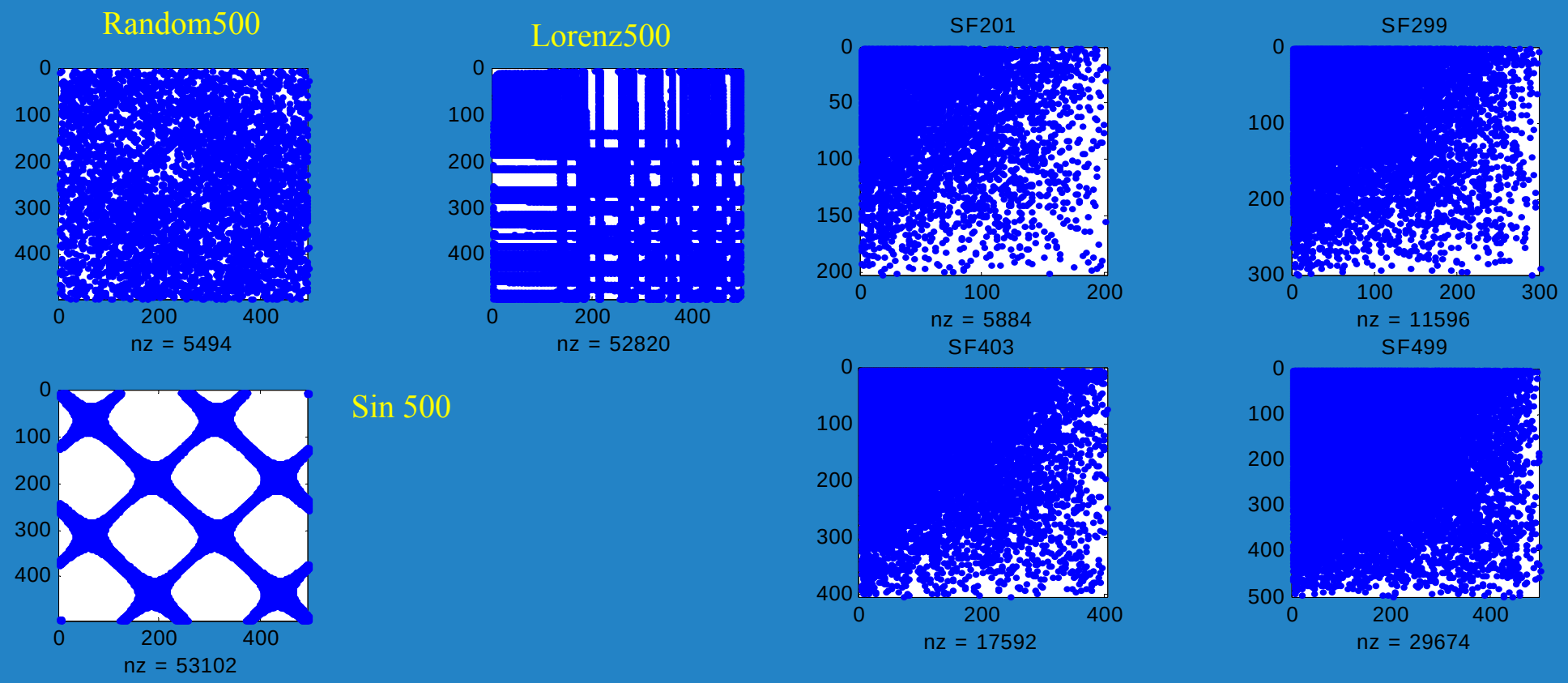
Preliminary results

Scale Free networks with Pajek

SF n° nodes of the strong component	Initial n° of nodes	Max n° of lines	Average degree of vertices	Initial E-R n° of nodes	Initial Probabiliy of lines	alpha
SF201	210	9339	50	10	0.9	0.25
SF299	305	12105	75	10	0.9	0.25
SF403	410	17696	75	10	0.9	0.25
SF503	510	26410	100	10	0.9	0.25

Preliminary results

Adjacency matrices



- pr= 1/10 of maximum distance
- R=3p

$$p = \frac{1}{10}\max\left(\frac{y(t) - \min_t(y(t))}{\max_t(y(t)) - \min_t(y(t))}\right)$$

Preliminary results

$G := (V, E)$ graph with n vertices,
V=set of vertexes, E=set of edges

Degree centrality of v

$$C_D(v) = \frac{\deg(v)}{n-1}$$

Degree centrality of G

$$C_D(G) = \frac{\sum_{i=1}^{|V|}[C_D(v_m) - C_D(v_i)]}{n-2}$$

v_m is the node with the maximum centrality in G, similarity with star structure

Betweenness centrality of v

$$C_B(v) = \sum_{\substack{s,t \in V \\ s,t \neq v}} \frac{\vartheta_{st}(v)}{\vartheta_{st}}$$

$\vartheta_{st}(v)$ is the number of shortest paths passing across v

ϑ_{st} is the total number of shortest paths

Mean distance of G

The mean of all the shortest paths between any couple of vertexes

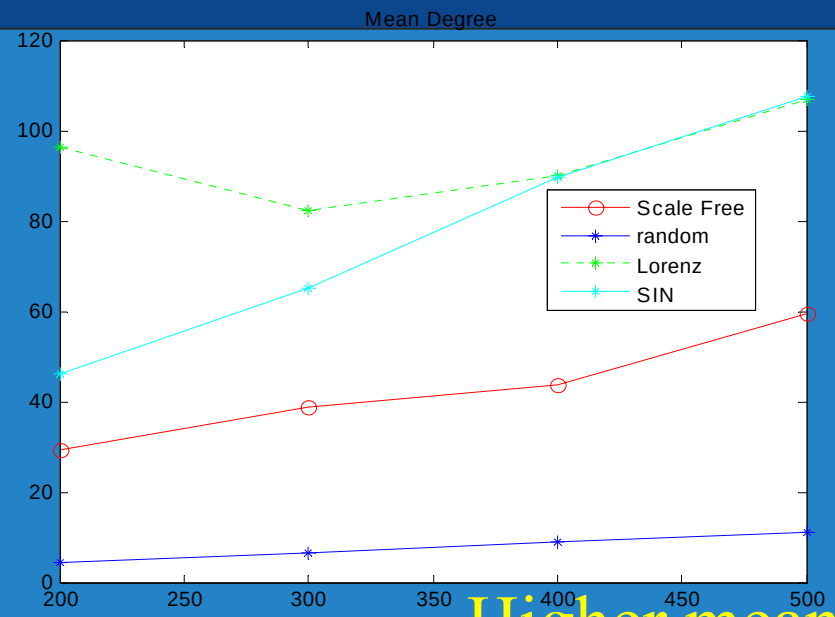
Diameter of G

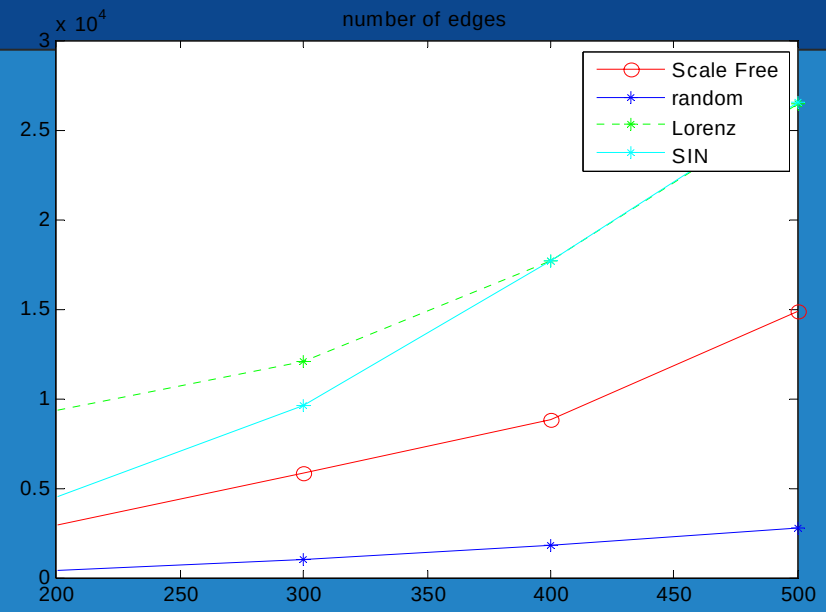
The maximum of all the shortest paths between any couple of vertexes

NEST Pathfinder

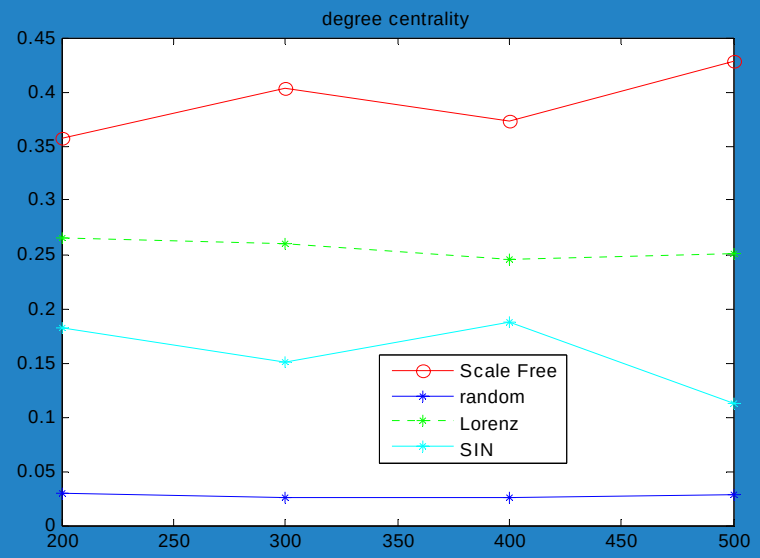
Than Made

Preliminary results



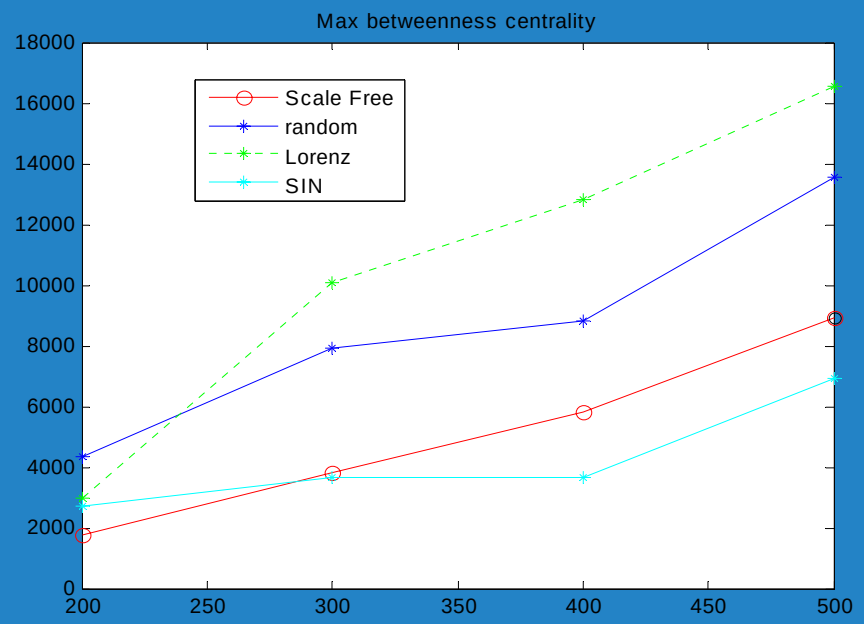


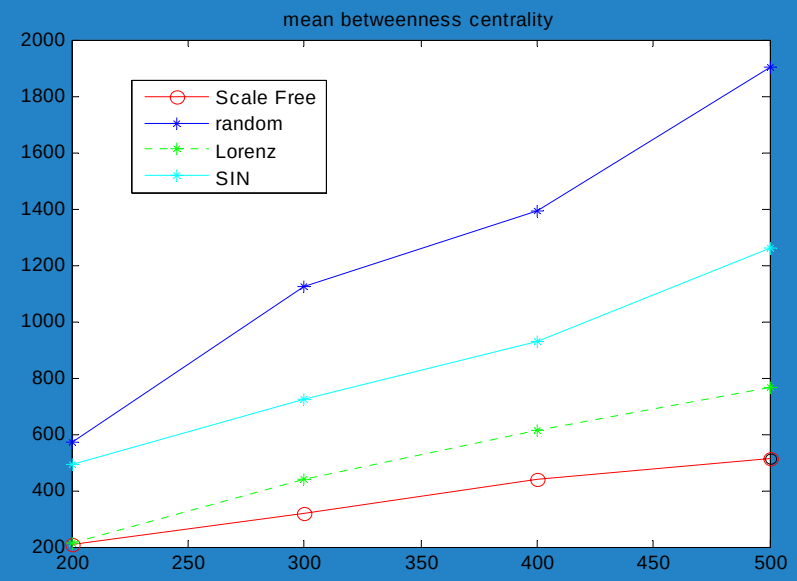
Higher mean degree and number of edges



Lower degree centrality=> less similar to star structure

Preliminary results

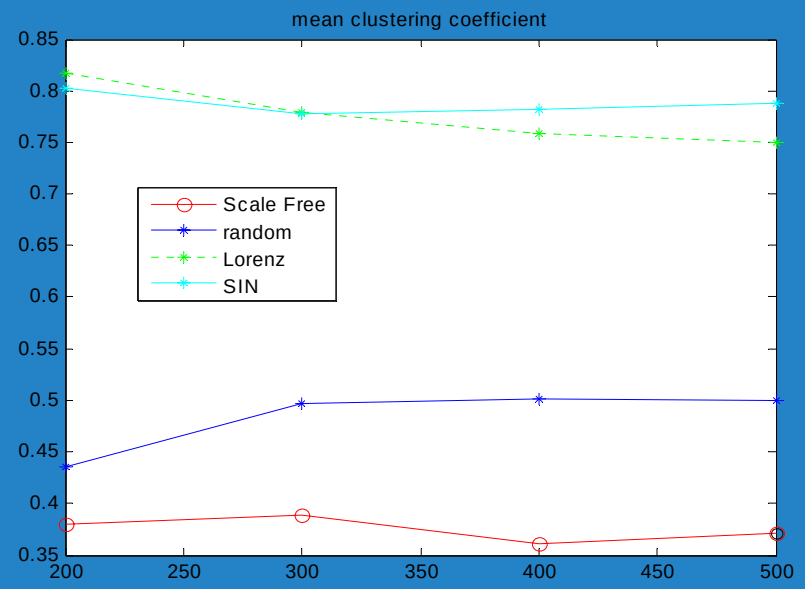


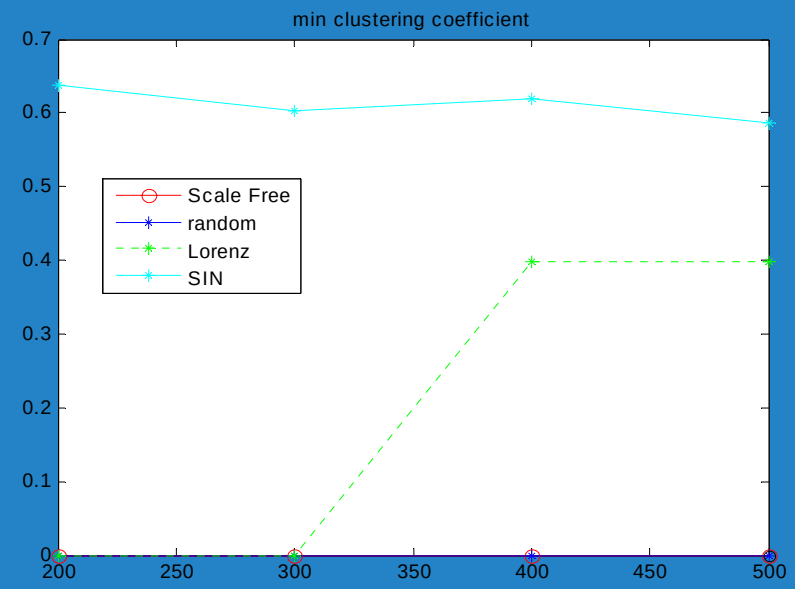


Higher max and mean betweennes centrality

=> higher percentage of the minimum paths cross the vertexes

Preliminary results



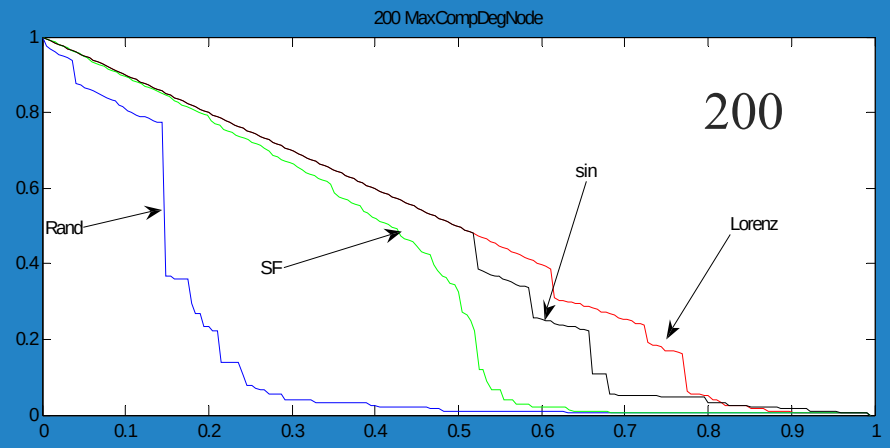


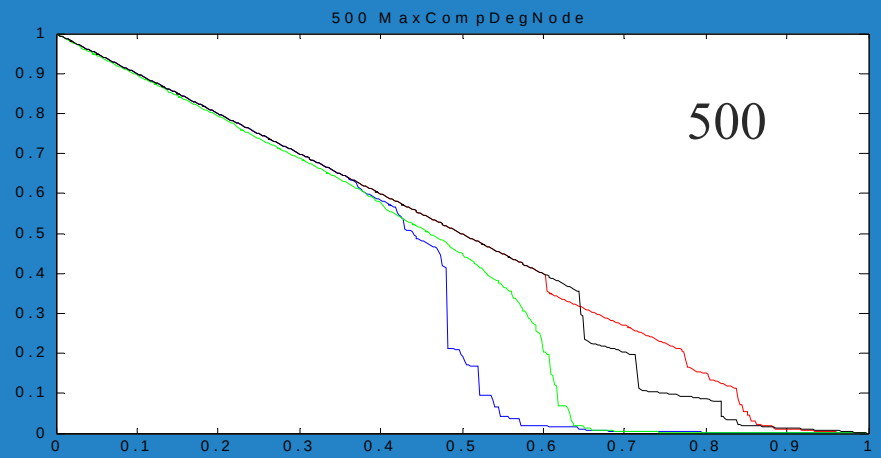
Higher mean and min clustering coefficient than random and scale free => the neighbors of the nodes are better connected

Vulnerability

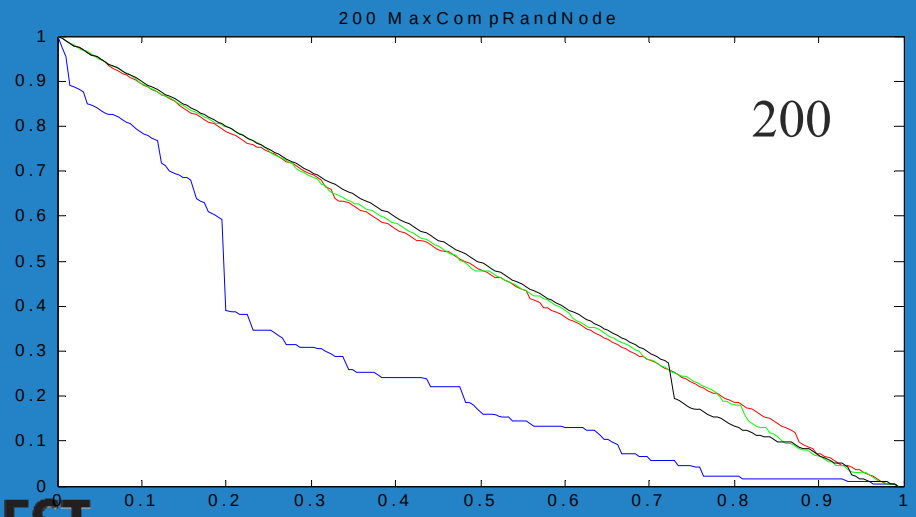
Size of the giant component
Percentage of attacked nodes

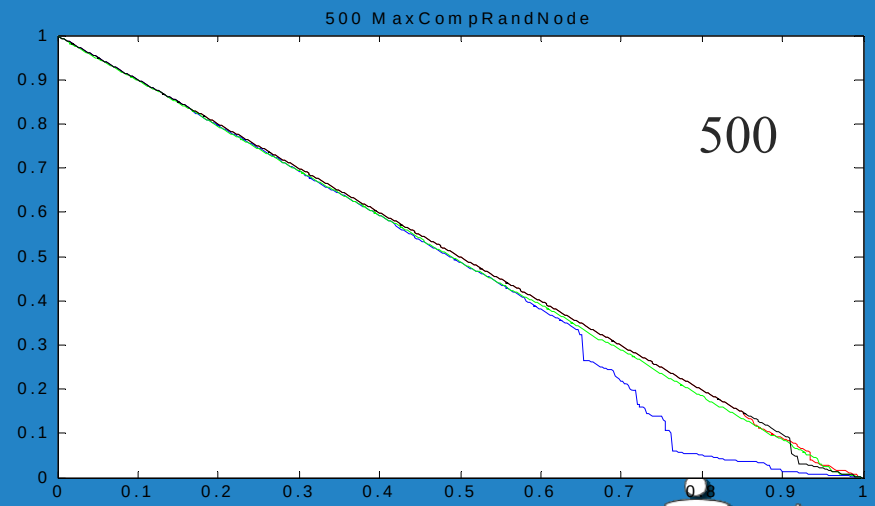
degree attacks





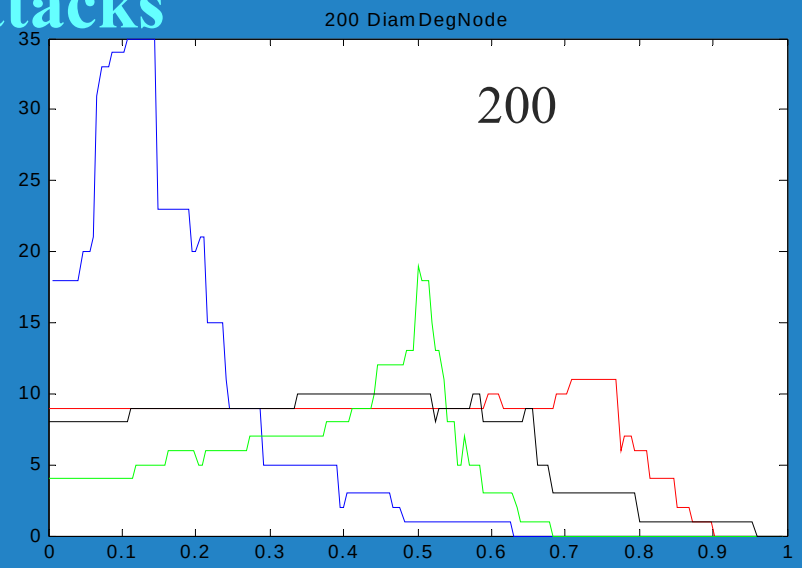
random attacks

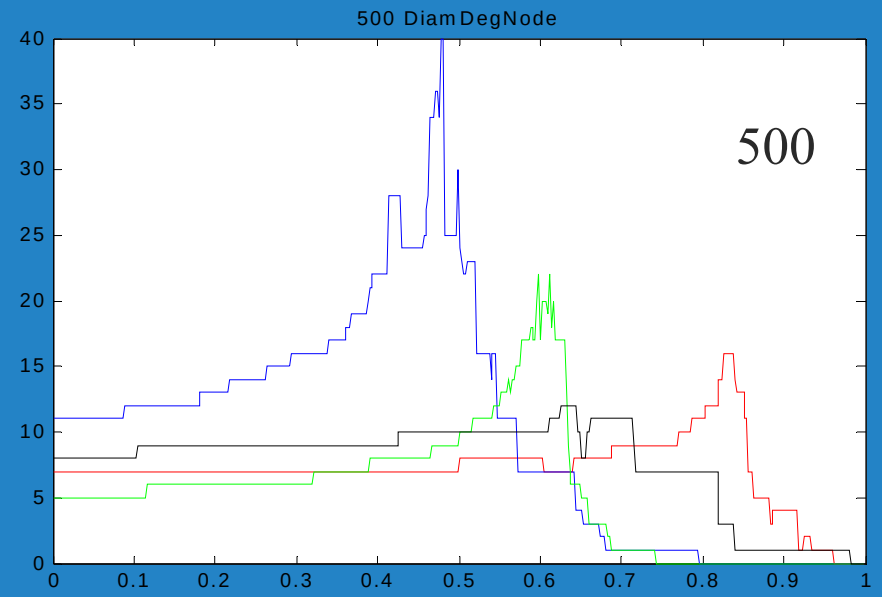




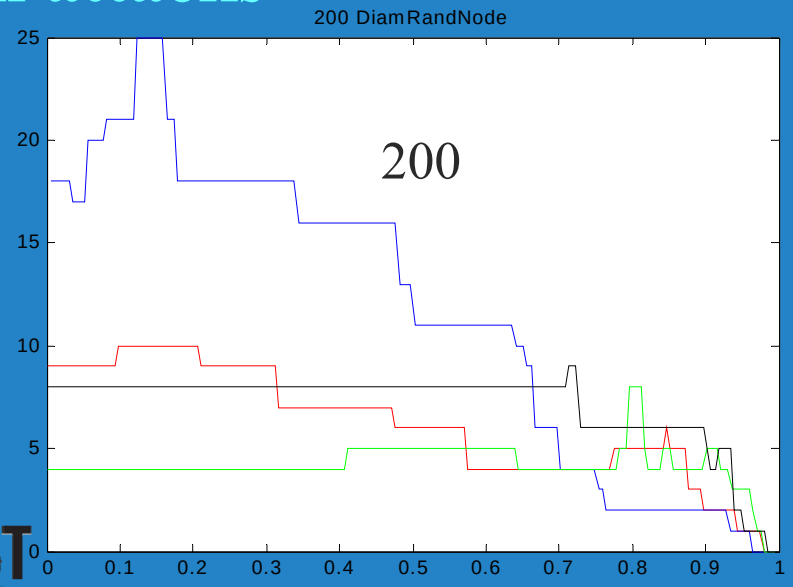
diameter
Percentage of attacked nodes

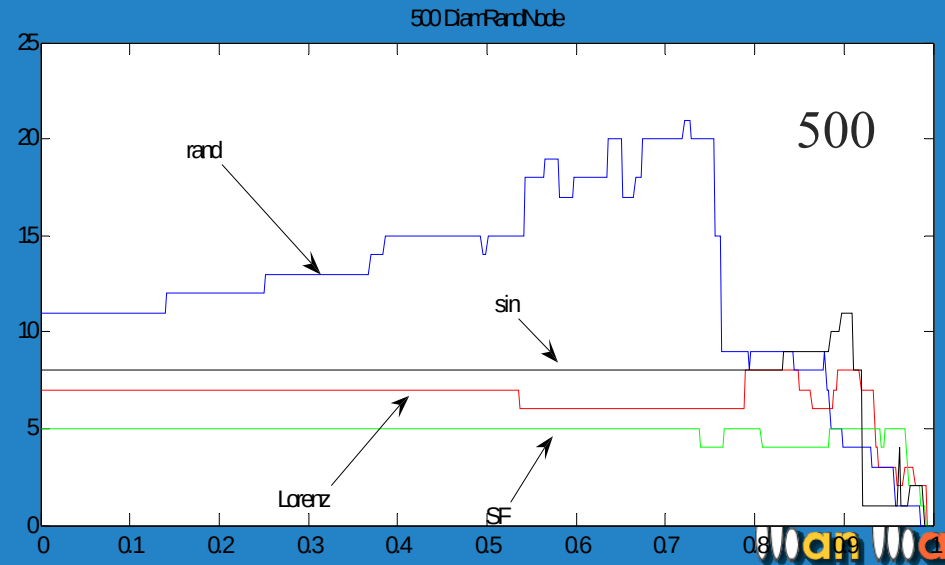
degree attacks





random attacks



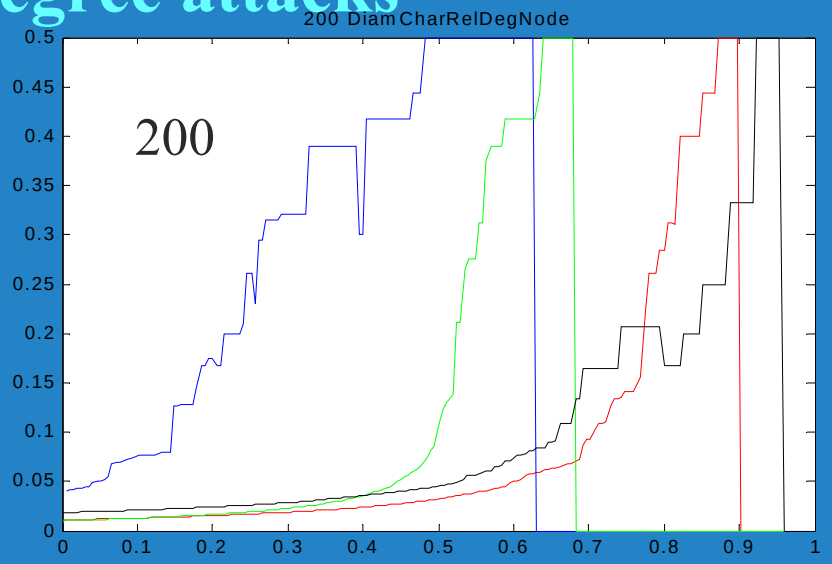


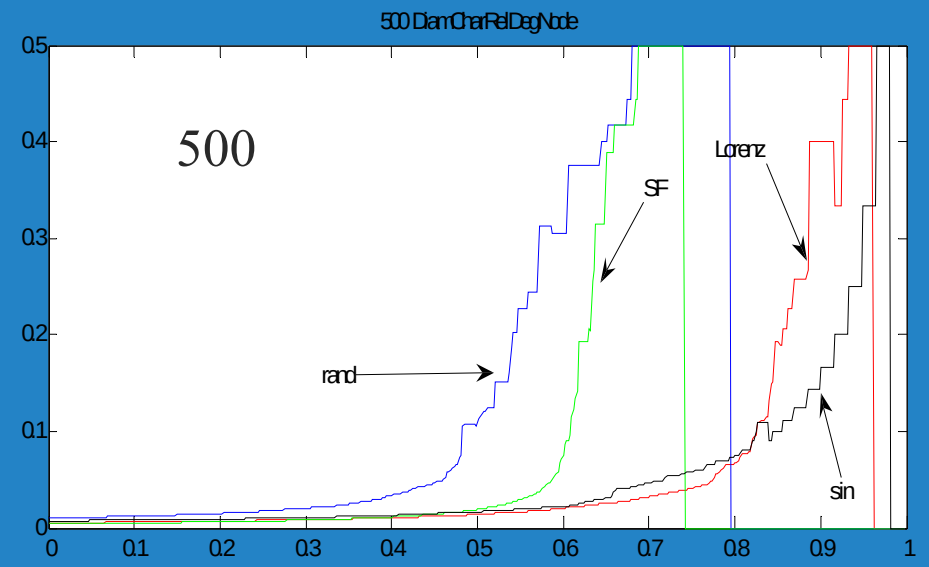
NEST Pathfinder

Man Made

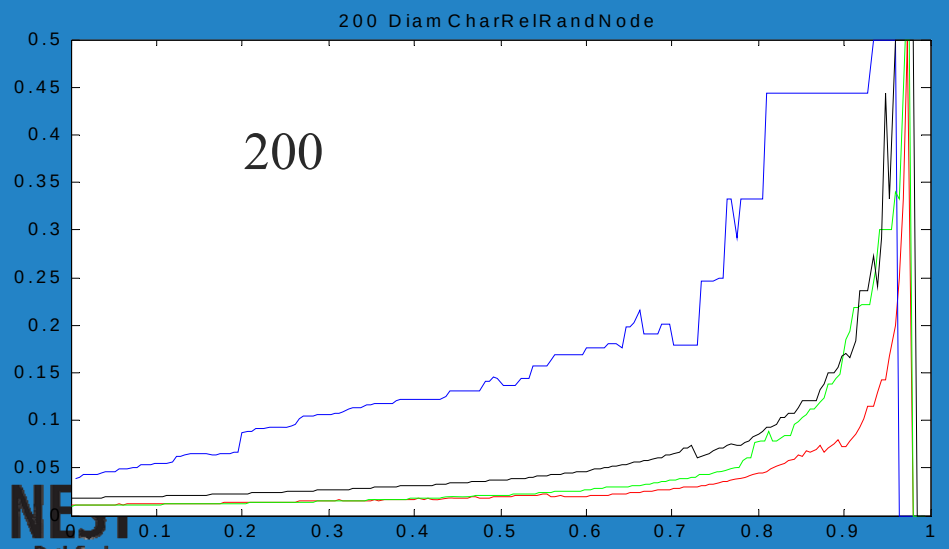
Characteristic length of giant component
Normalize with the size of giant component

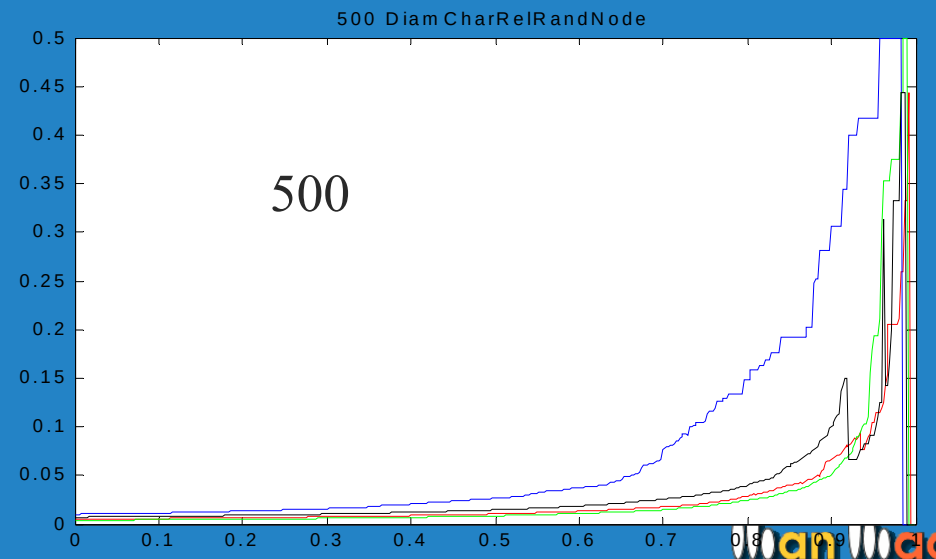
degree attacks





random attacks





Reproductive number R_0

The basic reproduction number (or rate) R_0 of an infection is the mean number of secondary cases infected by a single case in a population without immunity or control abilities.

$k_i =$ degree of vertex i, <>= mean

r = probability of transmission to neighbours

Connectivity parameter K

$$R_0 = r\frac{\sum_i k_i(k_i-1)}{\sum_i k_i} = r\frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle} > 1$$

$$r > \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}$$

$R_0 > 1$ The number of people infected continuously increases

$R_0 < 1$ The illness does not propagate

$$\langle k^2\rangle >> \langle k\rangle$$

r small and K small

NEST Pathfinder

Than Made

Connectivity parameter

$$R_0 = r\frac{\sum_i k_i(k_i - 1)}{\sum_i k_i} = r\frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1,$$

$$\text{if } r = 1, R_0 > 1 \Leftrightarrow \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

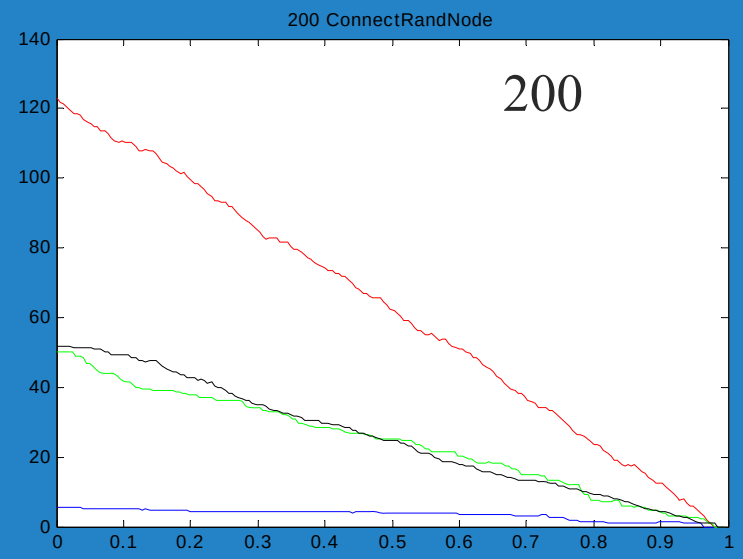
$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

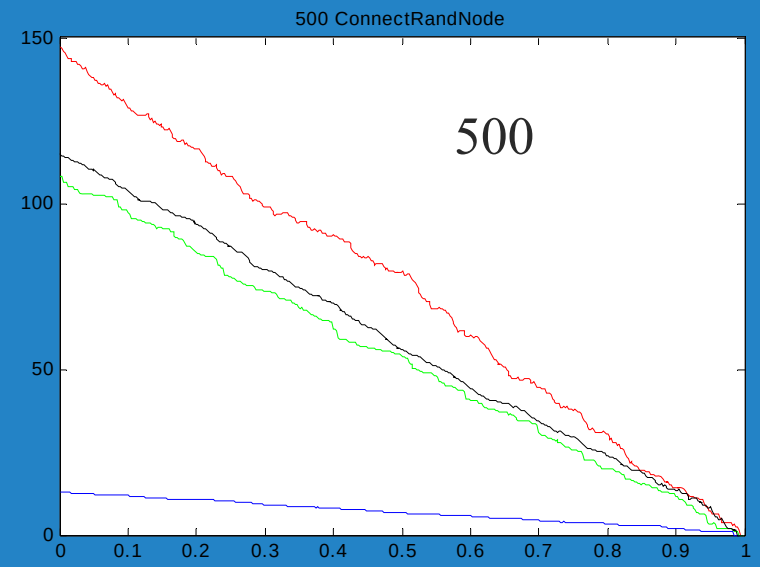
percolation threshold

The value of 2 is the criterion of the percolation threshold when the giant component disappears and the network becomes a set of small clusters.

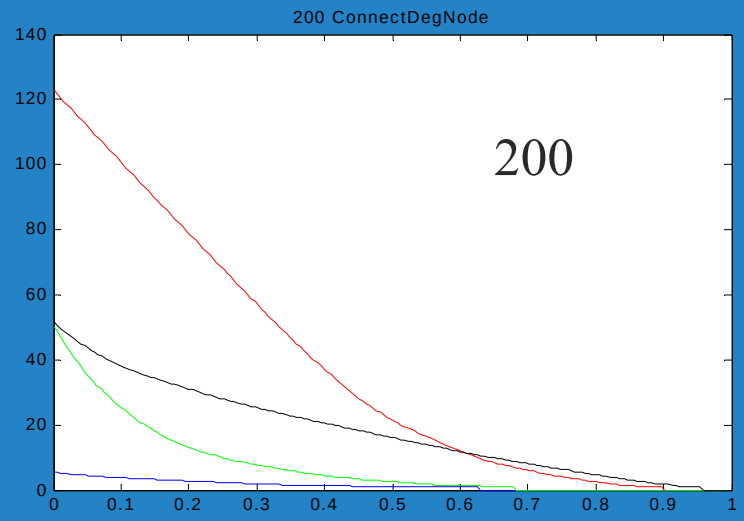
Connectivity parameter

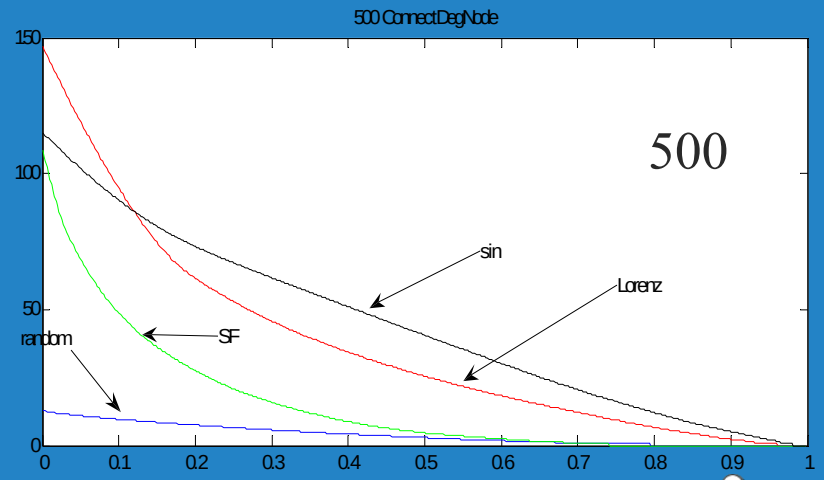
random attacks





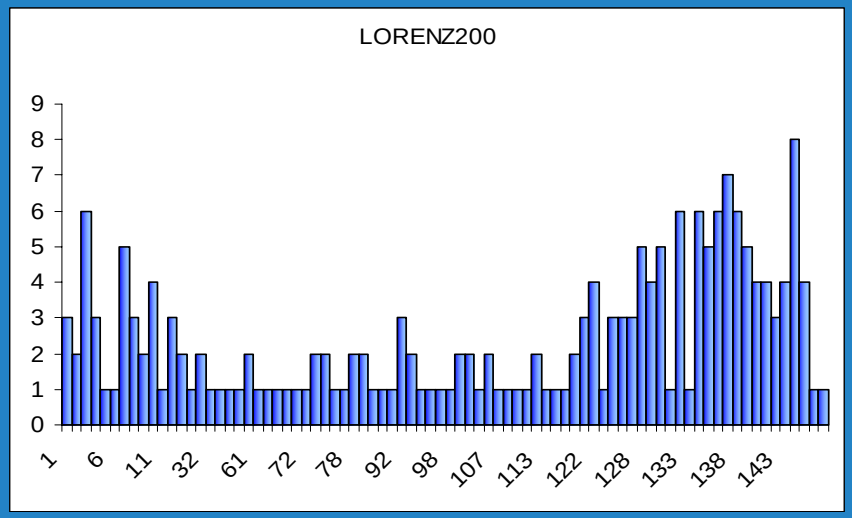
degree attacks

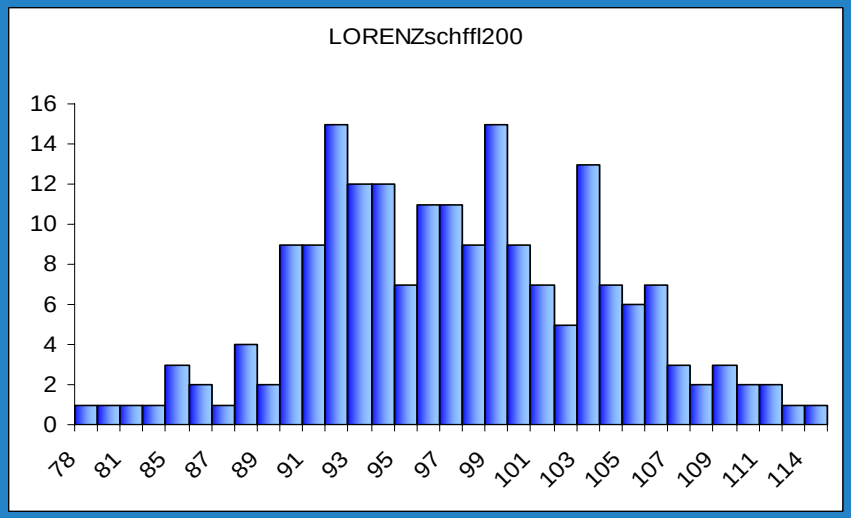




Instead of its structure, the robustness of Lorenz network can be due to the huge number of edges.
To check this aspect, we considered the network generated by a Lorenz shuffled adjacency matrix

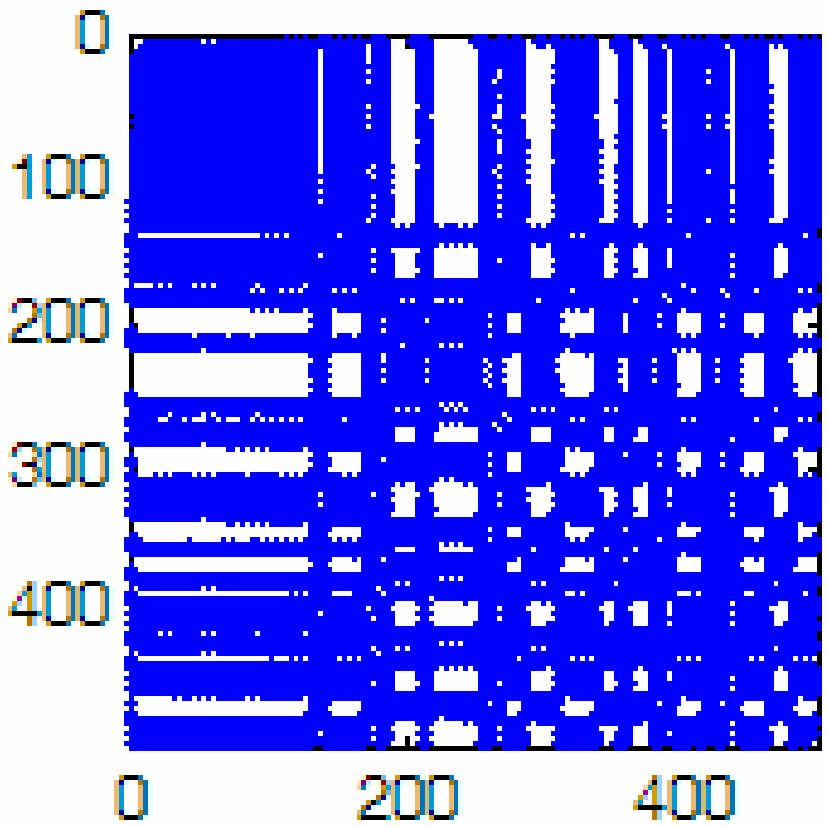
Lorenz shuffled network is similar to a regular graph in which all the nodes have the same degree



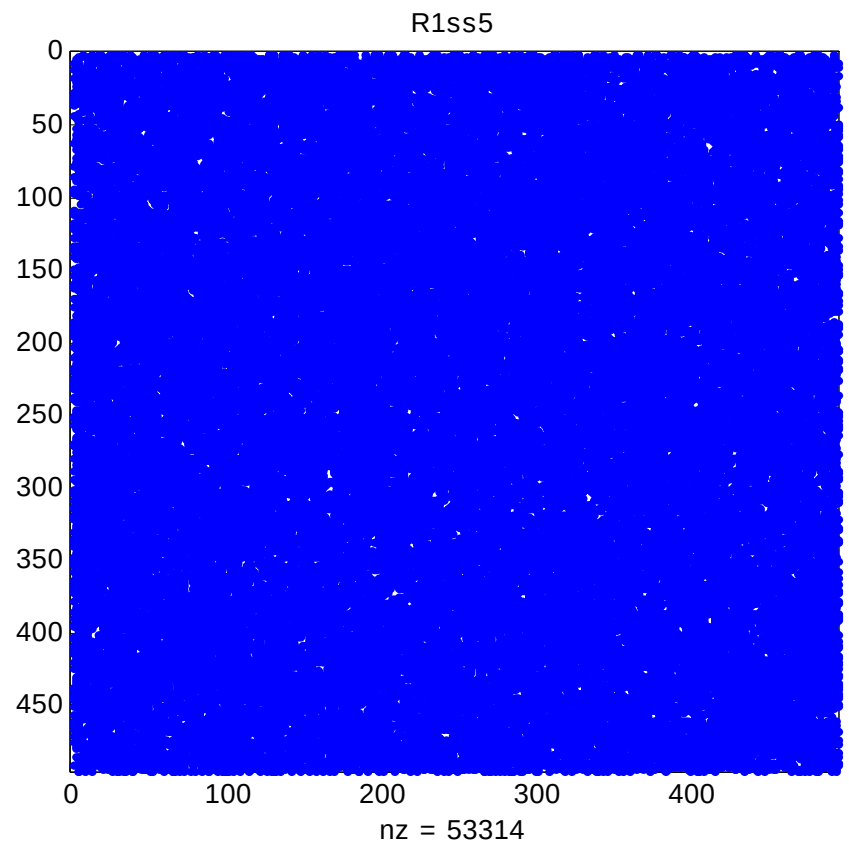


1 147 78 114

Lorenz 500

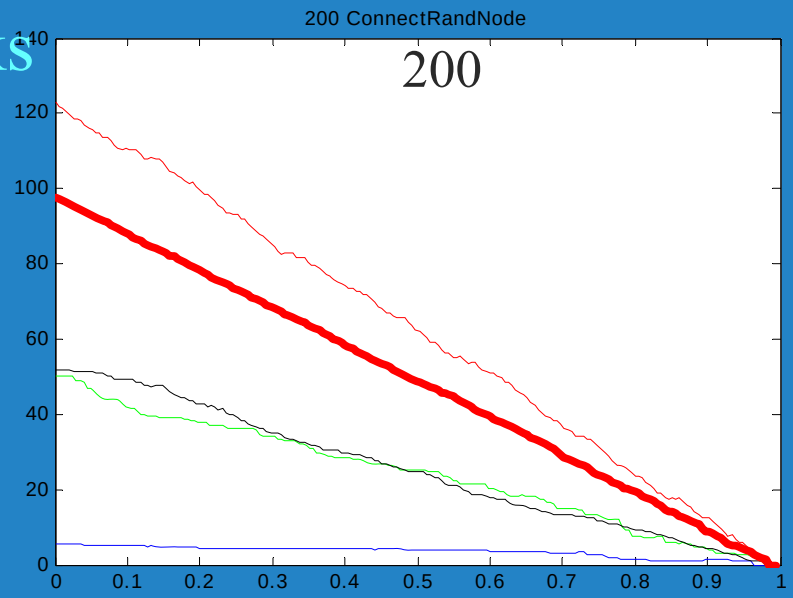


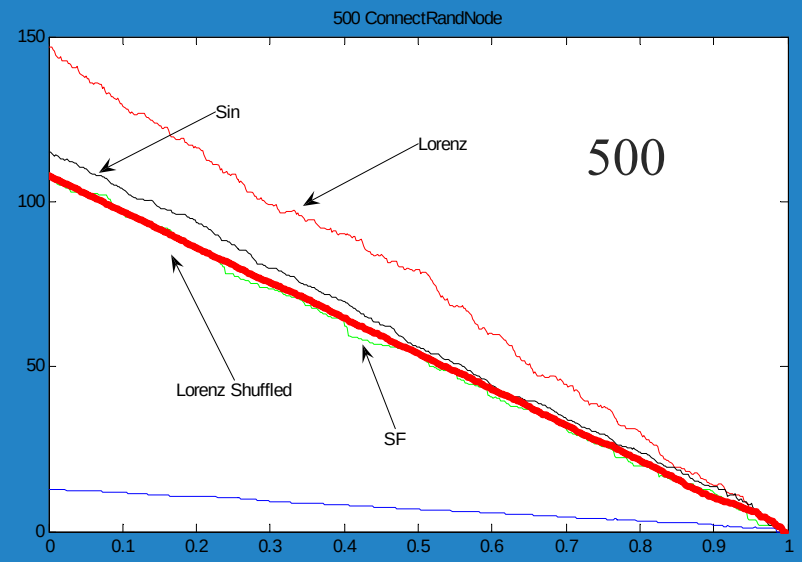
Lorenz shuffled 500



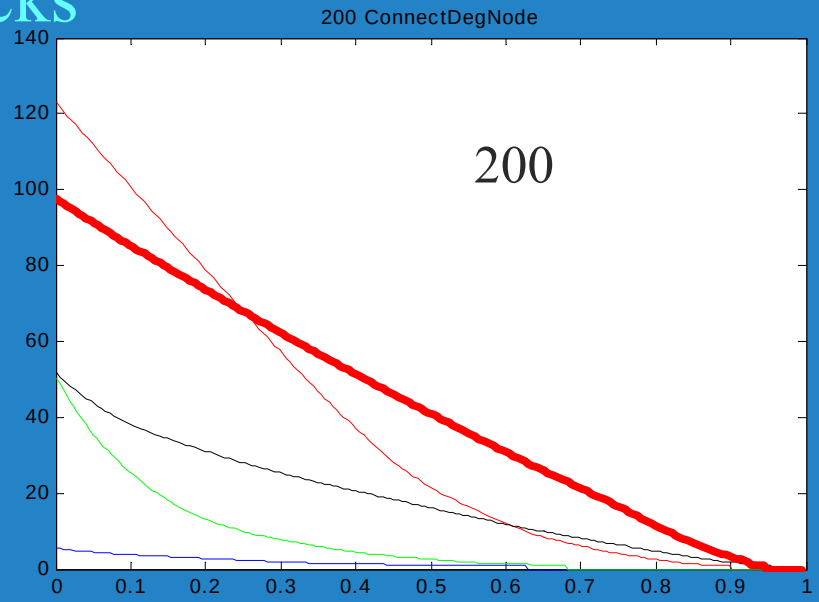
Connectivity parameter

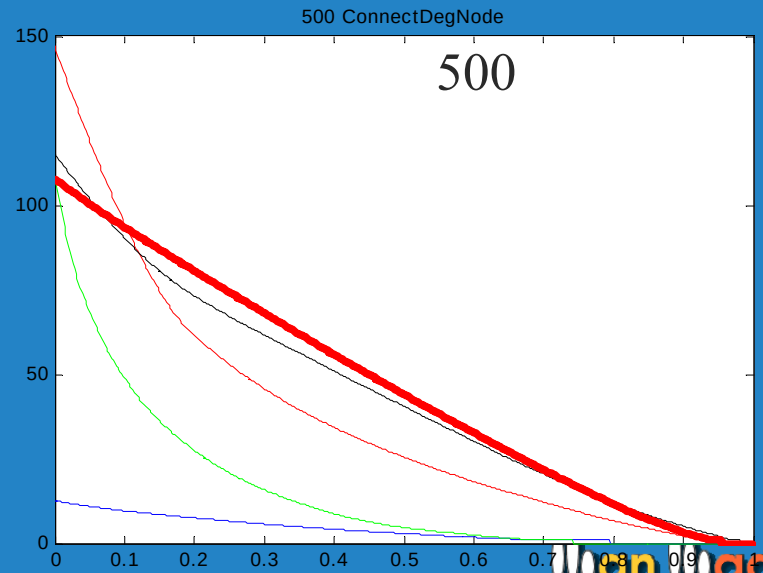
random attacks





degree attacks





NEST Pathfinder

Man Made

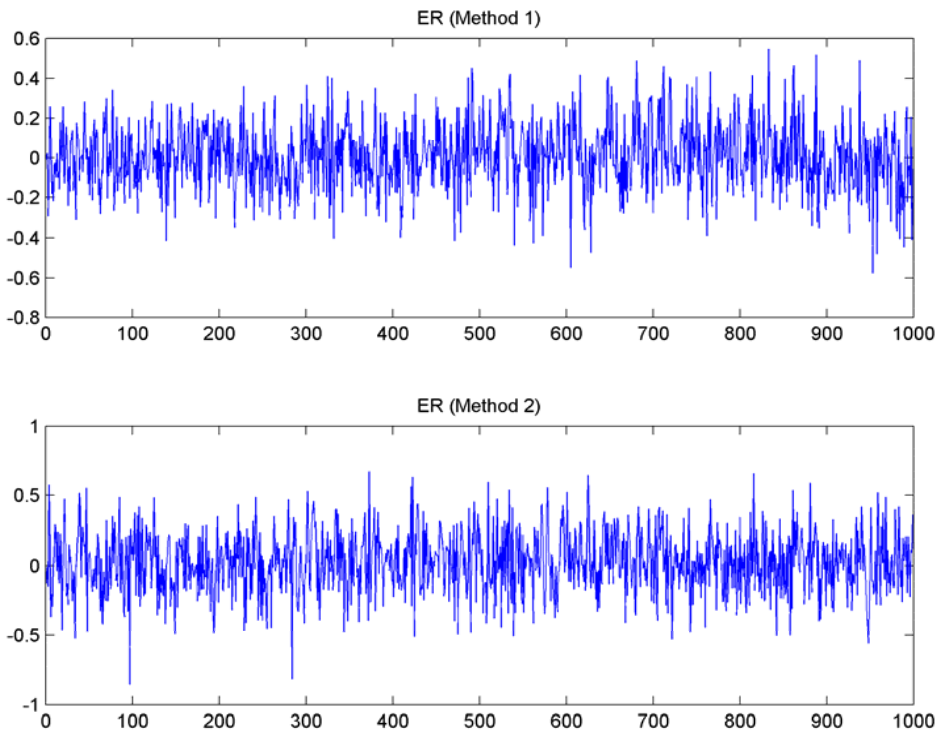
Preliminary results

Part B: From complex networks to time series

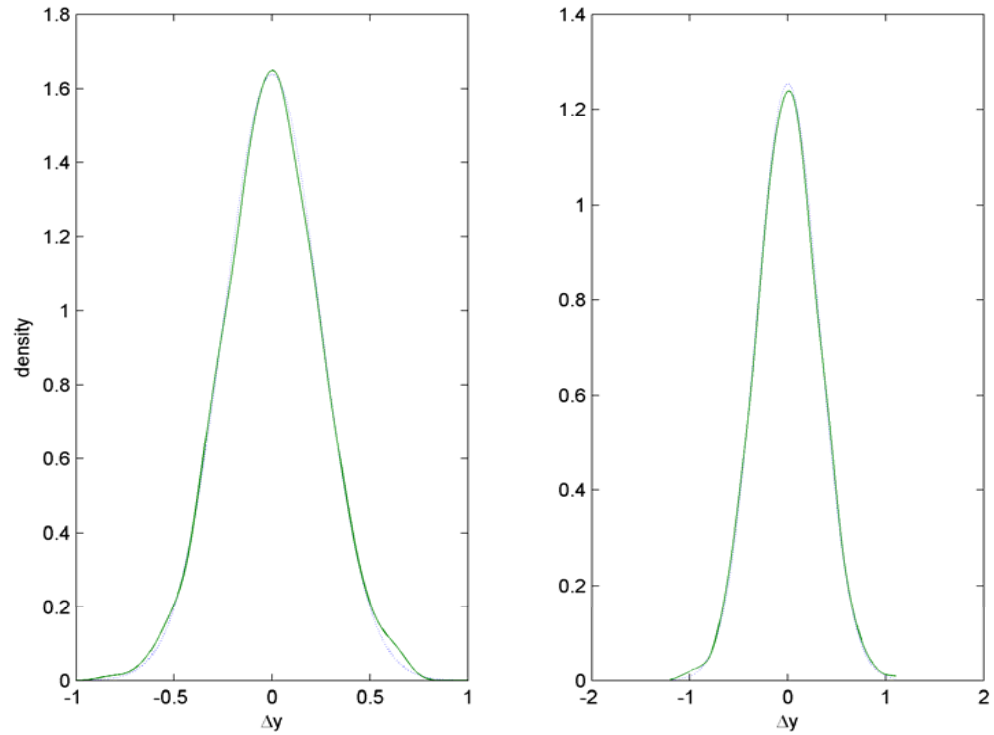
Gaussian and long tail probability distribution functions

Gaussian probability distribution of Erdos-Reny (ER) networks

E-R network 1000 nodes



Probability distribution function



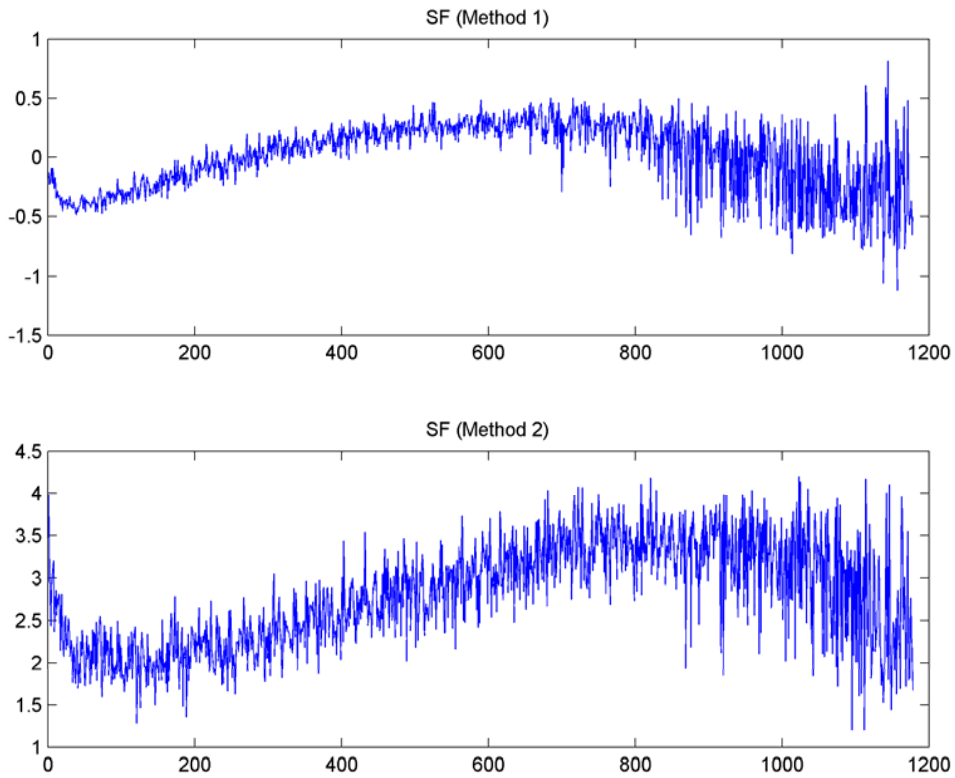
Method 1 → α= 2

Method 2 → α= 2

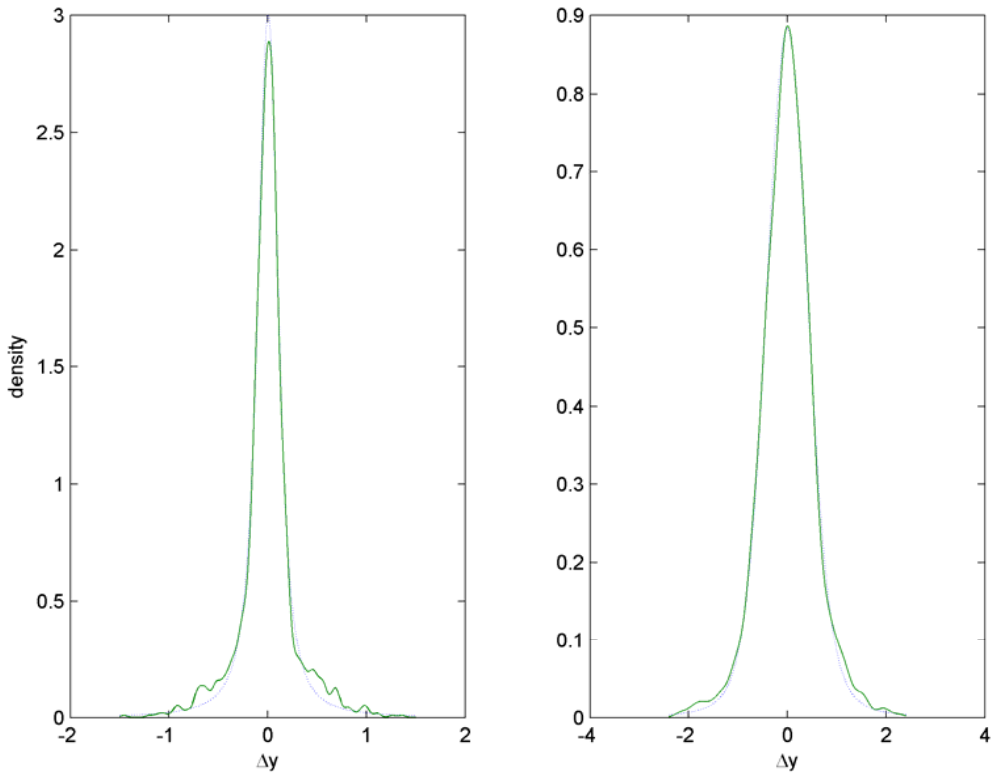
Gaussian (α = 2), Cauchy (α=1), Levy (α=1/2) and Dirac (α→0)

Fat tail probability distribution of Scale Free (SF) networks

SF network 1000 nodes



Probability distribution function



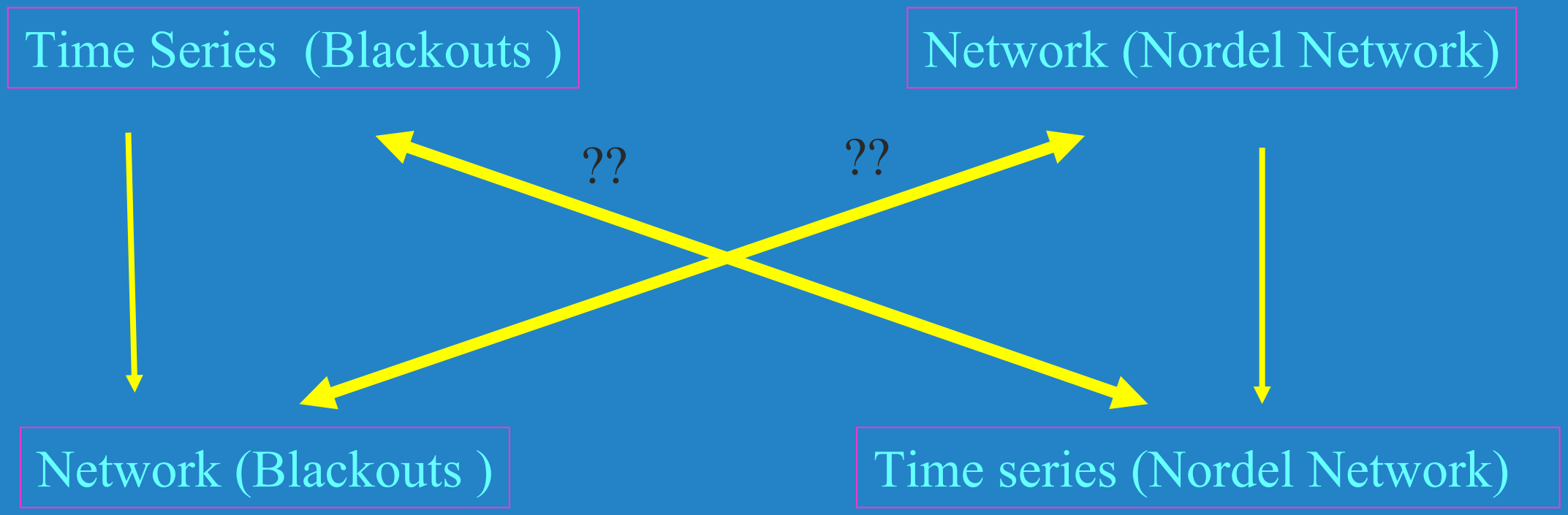
Method 1 → α= 1.2

Method 2 → α= 1.7

Conclusions

- A new method to move from Time Series (TS) to Complex Networks and *viceversa*
- Chaotic TS provide a deterministic law for evolving networks (Chaotic Networks)
- Lorenz networks properties: Connectivity and Diffusion
- ER networks produce TS with Gaussian pdf whereas SF networks produce TS with long tail pdf
- Problems from networks to TS: what does time means? Initial and final points?

Future work


Time Series (Blackouts)
Network (Nordel Network)
??
??
Network (Blackouts)
Time series (Nordel Network)