From complex networks to time series and *vice versa*

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Outline

1. Introduction

2. Methods and Approach

3. Preliminary results
   - From Time Series to Networks: Lorenz (Chaotic) versus Scale Free Networks
   - From Networks to Time Series: Gaussian and long tail probability distribution functions
Introduction

◆ Objectives:
  - Development of a general framework to transform complex networks into non-linear time series
  - Explore new research avenues that this dual vision of complex phenomena can provide us
Introduction

- **Existing Approaches:**

  - **From TS to NET**
    - Lacasa et al. 2008. From time series to complex networks: the visibility graph, PNAS 103, 4972-4975
    - Xu, Zhang and Small, 2009. Superfamily phenomena and motifs of networks induced from time series. PNAS 105, 19601-19605

  - **From NET to TS**
Methods and Approach

Recurrence Plot

Time series

Complex networks

Distance matrix
Multidimensional scaling/
Modal Analysis

Method 1: Multidimensional scaling (Hirata and Aihara, 2008)
Method 2: Modal analysis (Gutiérrez and Zaldivar, 2000)
Example: Complete process with Hénon map

1. \( G_i = \{ j : c(z_i, z_j) = 1 \} \)

2. \( w(i, j) = 1 - \frac{|G_i \cap G_j|}{|G_i \cup G_j|} \)

3. - Method 1: Multidimensional Scaling
   - Method 2: Modal Analysis
Methods and Approach: Examples

Random TS

Sin

Lorenz

Method 1

SF Network
Methods and Approach: Examples

Random TS

Sin TS

Lorenz TS

Method 2

SF Network
Methods and Approach: Examples

Network

Time Series

Network

Time Series
Preliminary results

Part A: From time series to networks

Lorenz versus Scale Free networks
**Preliminary results**

Uniform distribution on the unit interval $[0, 1]$

$$y = \text{rand}(1,500)$$

$$r = 28, \quad b = 8/3, \quad \sigma = 10$$

$$t = [0:5 \times \pi/500:5\pi]; \quad y = \sin(t);$$

\[
\begin{align*}
    \frac{dx(t)}{dt} &= \sigma (y(t) - x(t)) \\
    \frac{dy(t)}{dt} &= -y(t) + (r - z(t)) \cdot x(t) \\
    \frac{dz(t)}{dt} &= -b \cdot z(t) + x(t) \cdot y(t)
\end{align*}
\]
Reconstruction with:
$\Delta t=3$; $d_E=3$; $N=500$
## Preliminary results

### Scale Free networks with Pajek

<table>
<thead>
<tr>
<th>SF n° nodes of the strong component</th>
<th>Initial n° of nodes</th>
<th>Max n° of lines</th>
<th>Average degree of vertices</th>
<th>Initial E-R n° of nodes</th>
<th>Initial Probability of lines</th>
<th>alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF201</td>
<td>210</td>
<td>9339</td>
<td>50</td>
<td>10</td>
<td>0.9</td>
<td>0.25</td>
</tr>
<tr>
<td>SF299</td>
<td>305</td>
<td>12105</td>
<td>75</td>
<td>10</td>
<td>0.9</td>
<td>0.25</td>
</tr>
<tr>
<td>SF403</td>
<td>410</td>
<td>17696</td>
<td>75</td>
<td>10</td>
<td>0.9</td>
<td>0.25</td>
</tr>
<tr>
<td>SF503</td>
<td>510</td>
<td>26410</td>
<td>100</td>
<td>10</td>
<td>0.9</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Preliminary results

Adjacency matrices

Random500

Lorenz500

SF201

SF299

Sin 500

SF403

SF499

• \( pr = 1/10 \) of maximum distance
• \( R = 3p \)

\[
p = \frac{1}{10} \max_t \left( \frac{y(t) - \min_t y(t)}{\max_t y(t) - \min_t y(t)} \right)
\]
Preliminary results

\( G = (V, E) \) graph with \( n \) vertices, 
\( V = \) set of vertexes, \( E = \) set of edges

Degree centrality of \( v \)

\( C_D(v) = \frac{\text{deg}(v)}{n-1} \)

Degree centrality of \( G \)

\( C_D(G) = \frac{\sum_{i=1}^{\left| V \right|} \left[ C_D(v_m) - C_D(v_i) \right]}{n-2} \)

\( v_m \) is the node with the maximum centrality in \( G \), similarity with star structure

Betweenness centrality of \( v \)

\( C_B(v) = \sum_{s,t \in V, s,t \neq v} \frac{g_{st}(v)}{g_{st}} \)

\( g_{st}(v) \) is the number of shortest paths passing across \( v \)

\( g_{st} \) is the total number of shortest paths

Mean distance of \( G \)

The mean of all the shortest paths between any couple of vertexes

Diameter of \( G \)

The maximum of all the shortest paths between any couple of vertexes
Preliminary results

Higher mean degree and number of edges

Lower degree centrality => less similar to star structure

Scale Free
random
Lorenz
SIN

Mean Degree

Number of edges

Degree centrality
Preliminary results

Higher max and mean betweenness centrality

=> higher percentage of the minimum paths cross the vertexes
Higher mean and min clustering coefficient than random and scale free => the neighbors of the nodes are better connected
Vulnerability

Size of the giant component
Percentage of attacked nodes

degree attacks

random attacks
degree attacks

random attacks

diameter

Percentage of attacked nodes
Characteristic length of giant component
Normalize with the size of giant component

Degree attacks

Random attacks
Reproductive number $R_0$

The basic reproduction number (or rate) $R_0$ of an infection is the mean number of secondary cases infected by a single case in a population without immunity or control abilities.

$$R_0 = r \frac{\sum_i k_i (k_i - 1)}{\sum_i k_i} = r \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

$R_0 > 1$ The number of people infected continuously increases

$R_0 < 1$ The illness does not propagate

$\langle k^2 \rangle >> \langle k \rangle$ r small and K small
Connectivity parameter

\[ R_0 = r \frac{\sum k_i (k_i - 1)}{\sum k_i} = r \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1, \]

if \( r = 1, R_0 > 1 \iff \frac{\langle k^2 \rangle}{\langle k \rangle} > 2 \)

\[ \kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2 \]

The value of 2 is the criterion of the percolation threshold when the giant component disappears and the network becomes a set of small clusters.
Connectivity parameter

random attacks

degree attacks
Instead of its structure, the robustness of Lorenz network can be due to the huge number of edges.
To check this aspect, we considered the network generated by a Lorenz shuffled adjacency matrix

**Lorenz shuffled** network is similar to a regular graph in which all the nodes have the same degree
Lorenz 500

Lorenz shuffled  500

nz = 53314

nz = 53314
Connectivity parameter

random attacks

degree attacks

200 ConnectRandNode

500 ConnectRandNode

200 ConnectDegNode

500 ConnectDegNode
Preliminary results

Part B: From complex networks to time series

Gaussian and long tail probability distribution functions
Gaussian probability distribution of Erdos-Reny (ER) networks

E-R network 1000 nodes

Method 1 → $\alpha = 2$
Method 2 → $\alpha = 2$

Gaussian ($\alpha = 2$), Cauchy ($\alpha=1$), Levy ($\alpha=1/2$) and Dirac ($\alpha \to 0$)
Fat tail probability distribution of Scale Free (SF) networks

SF network 1000 nodes

Probability distribution function

Method 1 $\rightarrow \alpha = 1.2$
Method 2 $\rightarrow \alpha = 1.7$
Conclusions

- A new method to move from Time Series (TS) to Complex Networks and vice versa
- Chaotic TS provide a deterministic law for evolving networks (Chaotic Networks)
- Lorenz networks properties: Connectivity and Diffusion
- ER networks produce TS with Gaussian pdf whereas SF networks produce TS with long tail pdf
- Problems from networks to TS: what does time means? Initial and final points?
Future work

Time Series (Blackouts)       Network (Nordel Network)

??     ??

Network (Blackouts)       Time series (Nordel Network)