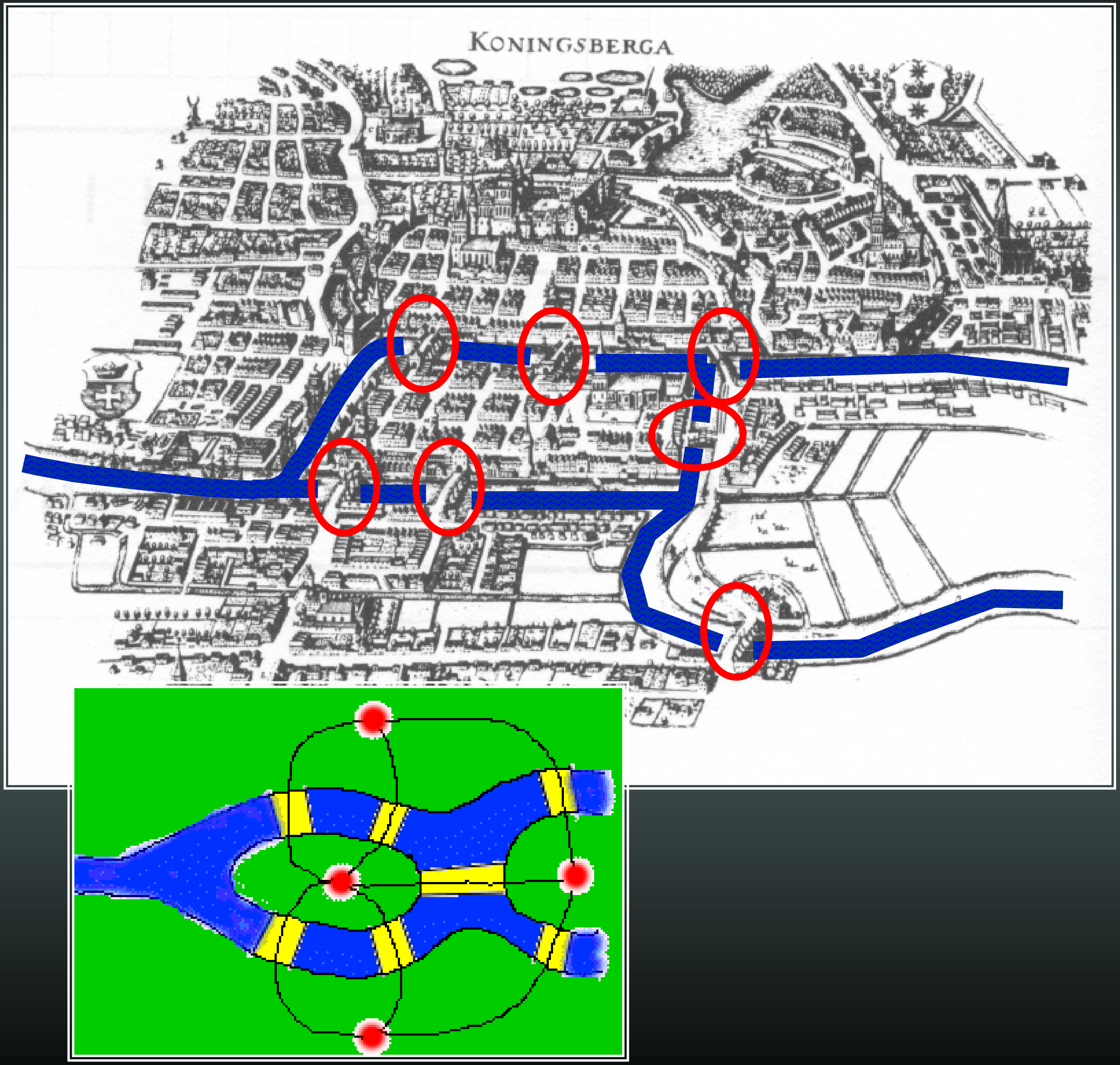
Structural and topological analysis of the European Power Grid

Case study

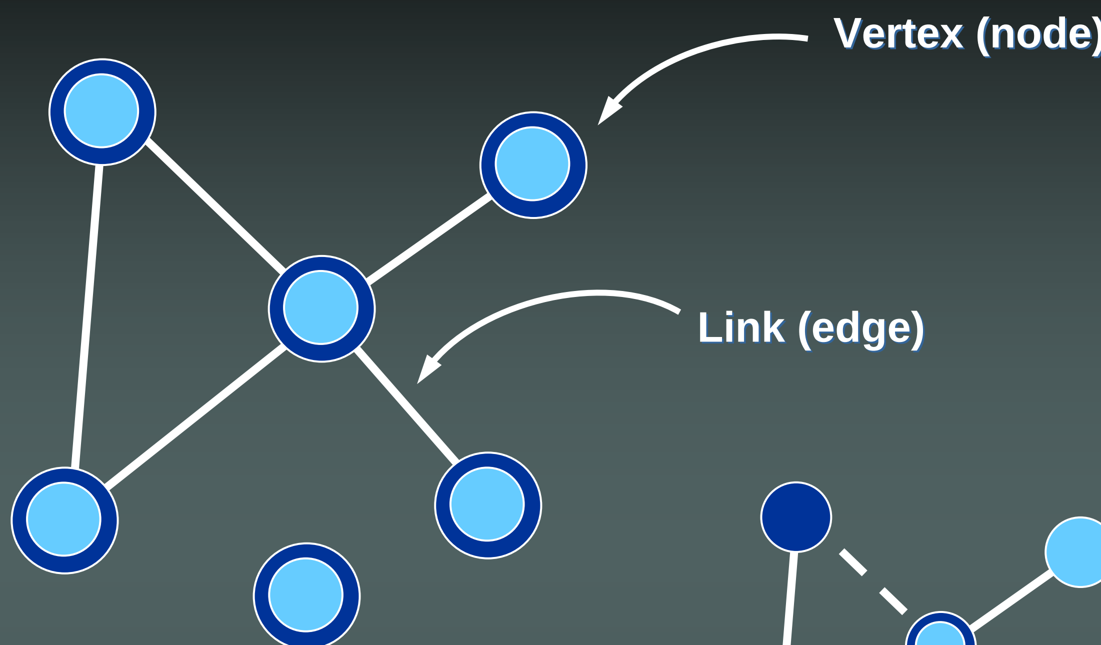
INTRODUCTION (1)

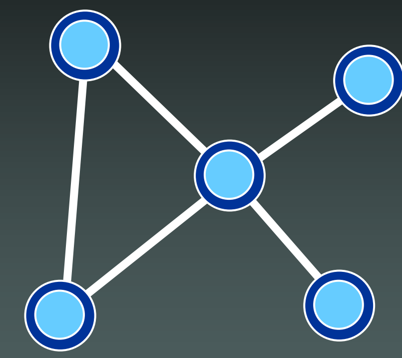


Leonhard Euler

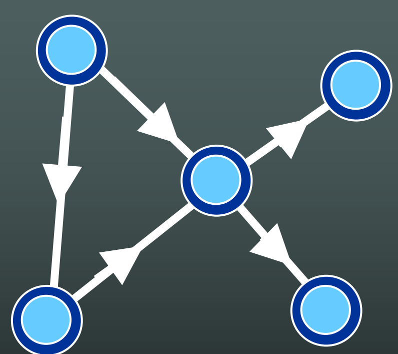
(1707-1783)

INTRODUCTION (2)

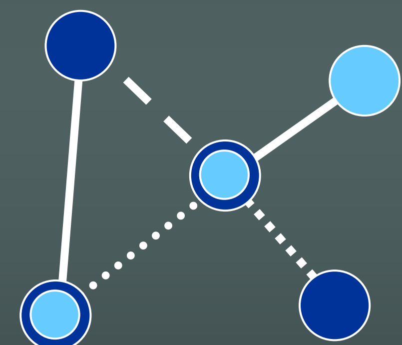




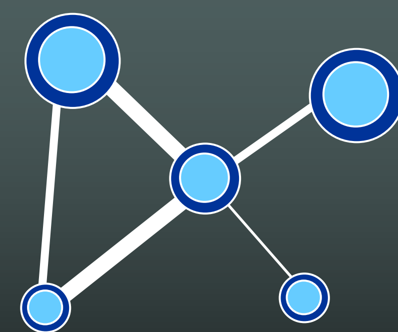
***Undirected* with 1 kind of vertex and 1 kind of edge**



***Directed* with one direction in every edge**

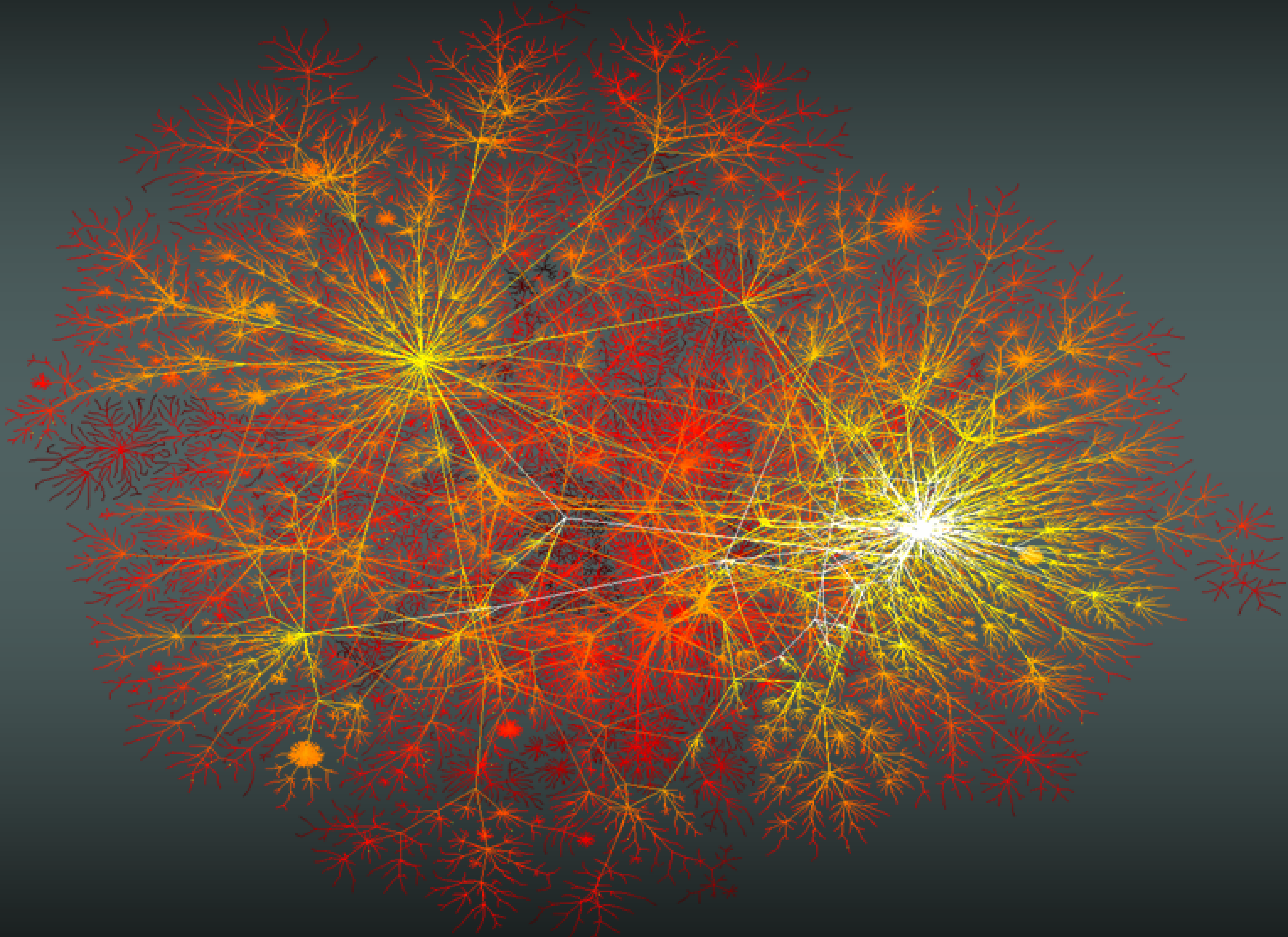


With a discrete number of kinds of nodes and edges



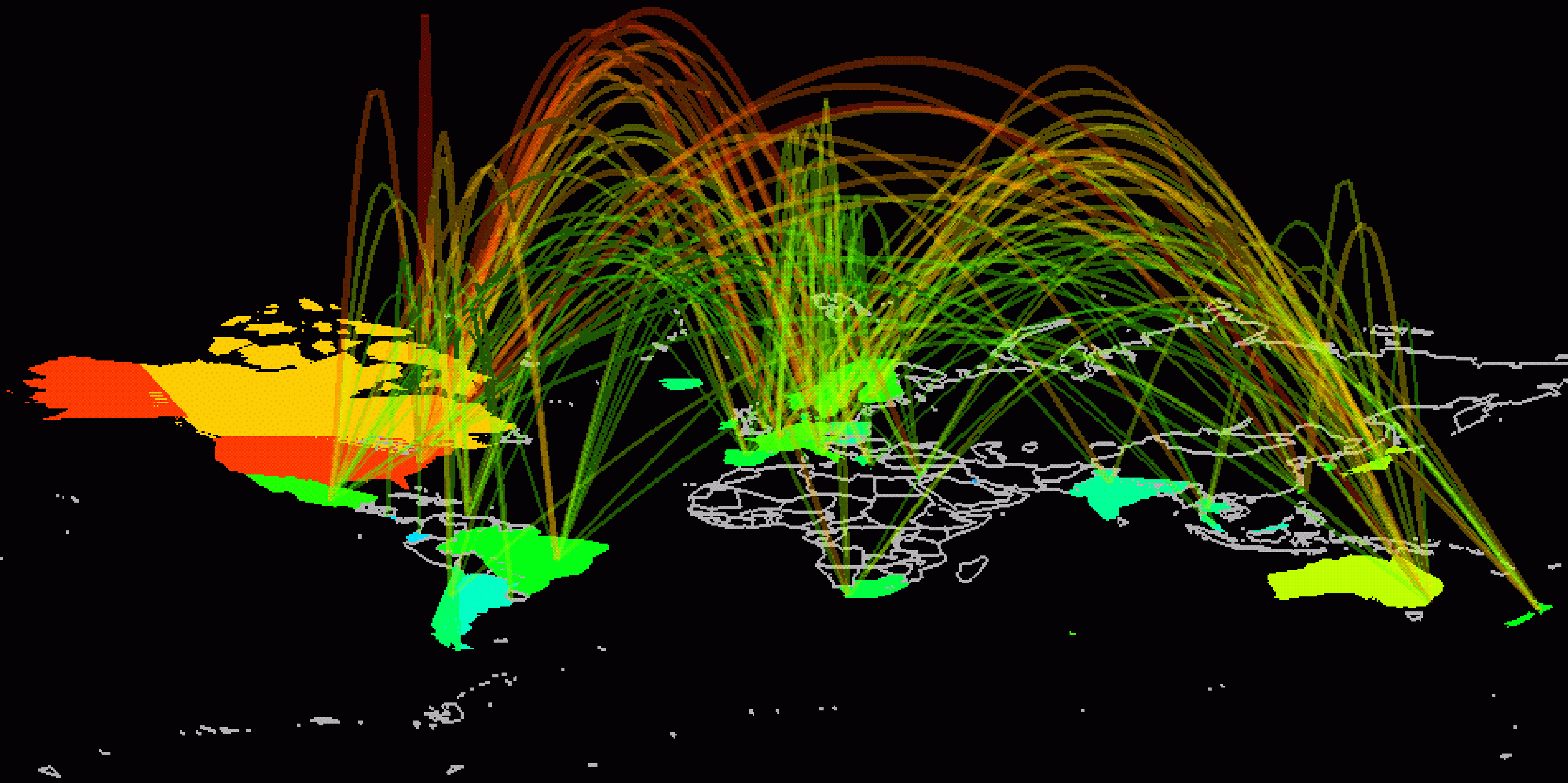
With different weights for nodes and edges

Example Networks (1)



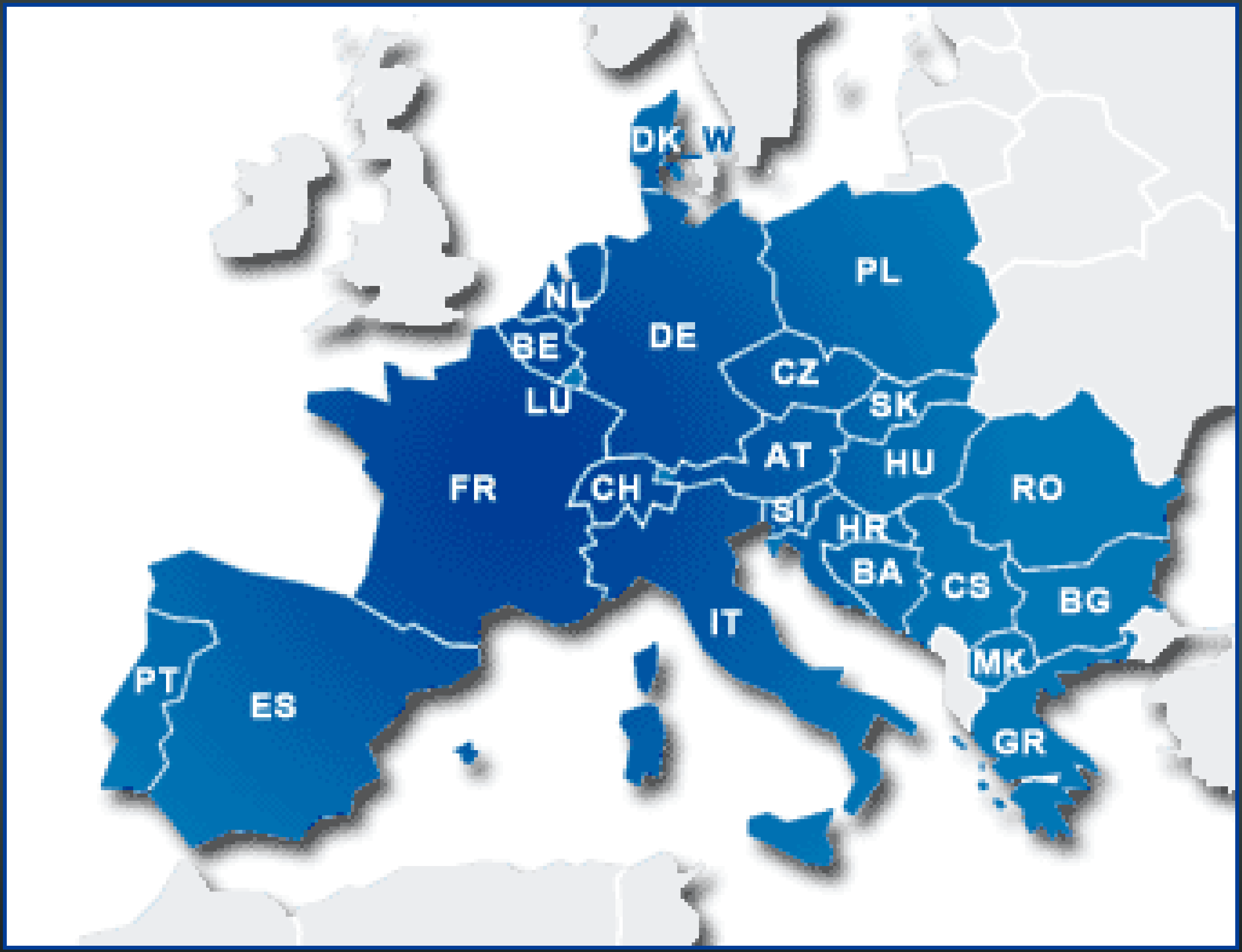
Internet at the autonomous system level (Internet Mapping Project, at http://www.cheswick.com/ches/map/gallery/index.html).

Example Networks (2)



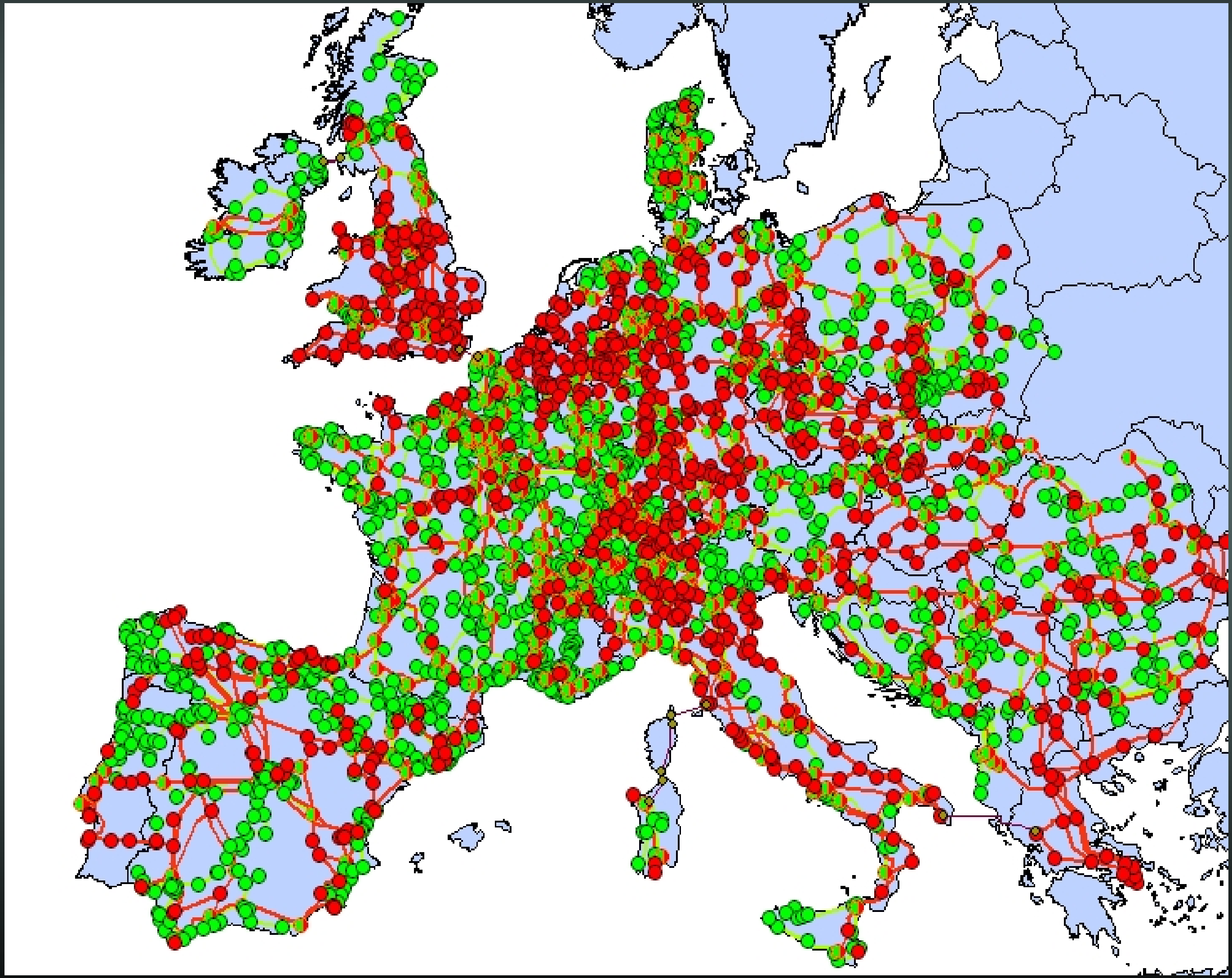
Telephone callings at a defined time step

EU Power grid (1)



Union for the Coordination of Transport of Electricity (UCTE)

EU Power grid (2)



Structural and Topological Measures

- Various centrality measures are defined in a graph G =(V,E), where V is the vertex set, E is the edge set, |V | =n, and |E| = m.
  - Authority and hub – HITS [1]
  - Degree Centrality [2] [3]
  - Betweenness Centrality [2] [4] [5]
  - Closeness Centrality [2] [6] [7]
  - Eigenvector Centrality – Pagerank [8] [9]
  - Information Centrality [10] [11]
  - Topological Centrality [12]

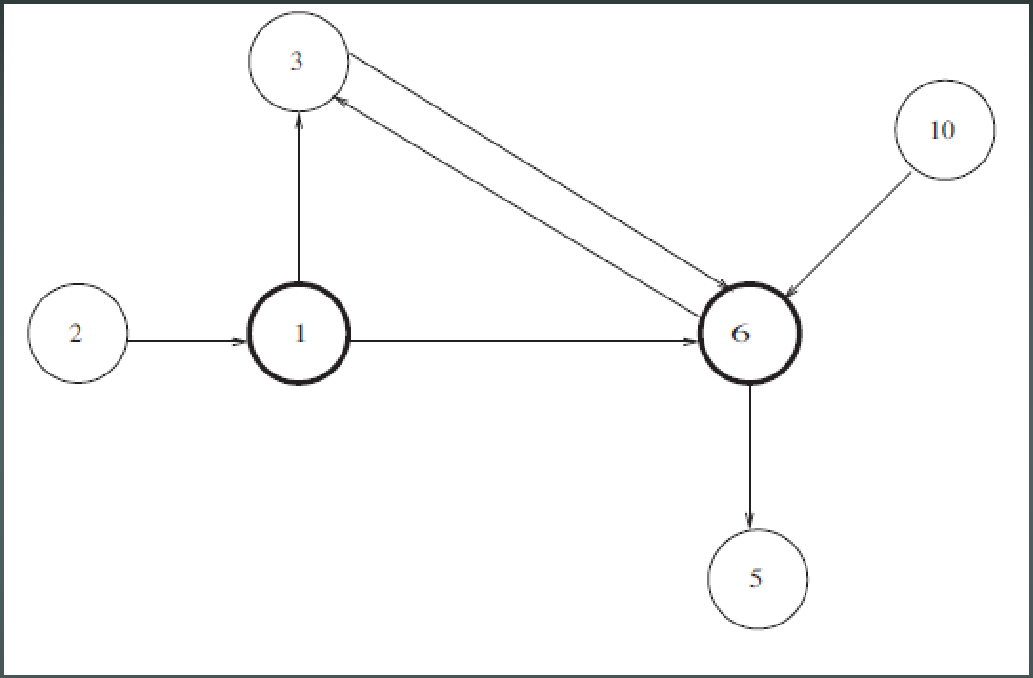
Authority and Hub – HITS [1]

- It reflects in-degree and out-degree characteristics of a certain node.
- The basic idea:
  - Good hub is linked to many authorities
  - A good authority is linked by many good hubs

$$\begin{cases} a(i) = \sum_{(j,i)\in E} h(j) \\ h(j) = \sum_{(i,j)\in E} a(i) \end{cases},$$

where a(x) and h(x) are the authority and hub of node x.

HITS example



$$x^T = (\,0 \quad 0 \quad .3660 \quad .1340 \quad .5 \quad 0\,),$$
$$y^T = (\,.3660 \quad 0 \quad .2113 \quad 0 \quad .2113 \quad .2113\,).$$

$$\text{Auth. ranking} = (\,6 \quad 3 \quad 5 \quad 1 \quad 2 \quad 10\,),$$
$$\text{Hub ranking} = (\,1 \quad 3 \quad 6 \quad 10 \quad 2 \quad 5\,).$$

Degree Centrality

- Based on the idea that more important nodes are more active, that is, they have more neighbours in the graph.
- May be used for finding the core nodes of a certain community.
- The problem is that this approach ignores the authority characteristics.

$$C_D(v) = \frac{deg(v)}{n-1}.$$

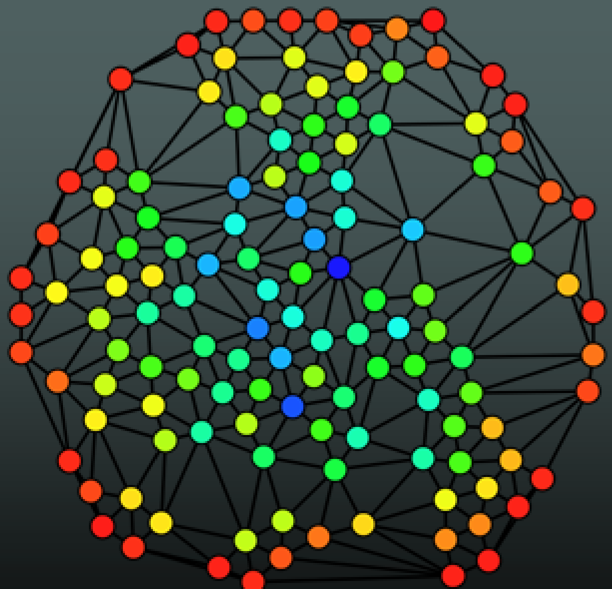
Betweenness Centrality (1)

- Describes the frequencies of nodes in the shortest paths between two indirectly connected nodes
- Based on the idea that if more nodes are connected via a node, then the node is more important
- Betweenness Centrality $C_B(v)$ for vertex v is:

$$C_B(v) = \sum_{\substack{s \neq v \neq t \in V \\ s \neq t}} \frac{\sigma_{st}(v)/\sigma_{st}}{(n-1)(n-2)},$$

Betweenness Centrality (2)

- Where σ_{st} is the number of shortest geodesic paths from s to t, and $\sigma_{st}(v)$ is the number of shortest geodesic paths from s to t that pass through a vertex v.



Closeness Centrality

- Describes the efficiency of the information propagation from one node to the other nodes.
- Based on the idea that if a node can quickly reach others, then the node is central.
- Can be regarded as a measure of how long it will take information to spread from a given vertex to other reachable vertices in the network.

$$C_c(v) = \frac{n-1}{\sum_{t \in V \setminus v} d_G(v,t)},$$

Eigenvector Centrality

- Describes the importance of nodes according to the adjacent matrix of a connected graph.
- It assigns relative scores to all nodes in the network based on the principle that connections to high-scored nodes contribute more to the score of a node than connections to low-scored nodes.

$$x_i = \frac{1}{\lambda} \sum_{j \in M(i)} x_j = \frac{1}{\lambda} \sum_{j=1}^{N} A_{i,j} x_j$$

Information Centrality (1)

- Describes nodes' influence on the *network efficiency* of information propagation.
- Network efficiency is defined by:

$$E_G = \frac{\sum_{i \neq j \in G} \epsilon_{ij}}{n(n-1))} = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}},$$

- where the efficiency ε_{ij} in the communication between two points *i* and *j* is equal to the inverse of the shortest path length d_{ij}

Information Centrality (2)

- The information centrality of a vertex *i* is defined as the relative drop in the network efficiency caused by the removal from *G* of the edges incident with *v*:

$$C_I(v) = \frac{\Delta E}{E} = \frac{E[G] - E[G'_v]}{E},$$

- where G'_v indicates a network by removing the edges incident with node *v* from *G*.

Topological Centrality (1)

- It reflects the topological positions of the nodes and edges in the network
- It can be used for discovering communities and building the backbone network
- *When a network is in the steady state, the topological centrality (TC) of a node is the ratio of its weight to the largest weight of nodes. The topological centers have the largest weight of node* 1. *The topological centrality of an edge is the ratio of its weight to the largest weight of node.*

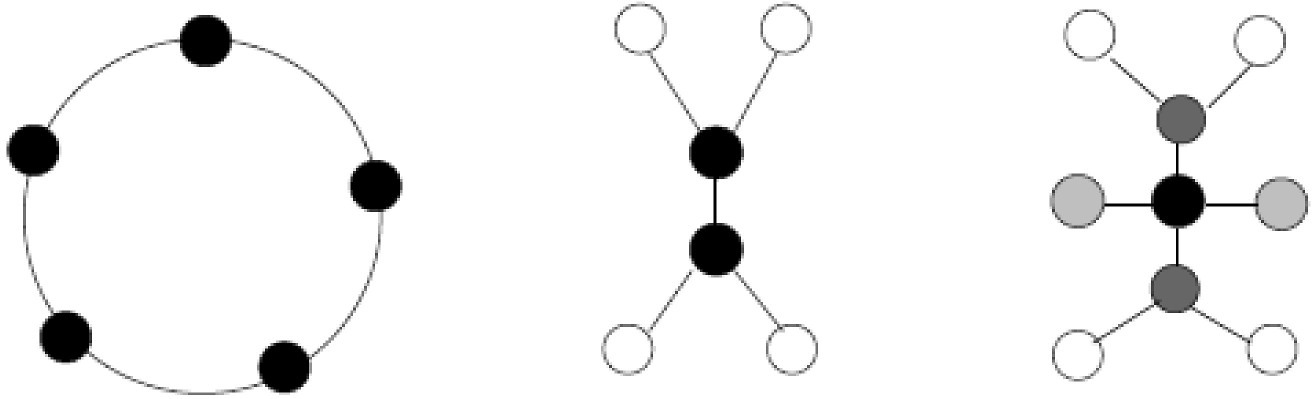
Topological Centrality (2)

- **Hypothesis 1.** *The topological centrality of a node is positively influenced by the topological centrality degrees of its neighbour nodes.*
    1. a node connecting to nodes with higher TC degrees gets higher TC degree; and,
    2. a node connecting to more nodes gets higher TC degree.
- **Hypothesis 2.** *If two nodes of an edge have higher TC degrees, then the edge has higher TC; and, if an edge has higher TC, then its two nodes also have higher TC degrees.*

  1. nodes closer to the topological center have higher TC degrees; and,

  2. edges closer to the topological center have higher TC degrees. These characteristics reflect that nodes with higher TC degrees

Topological Centrality - Examples



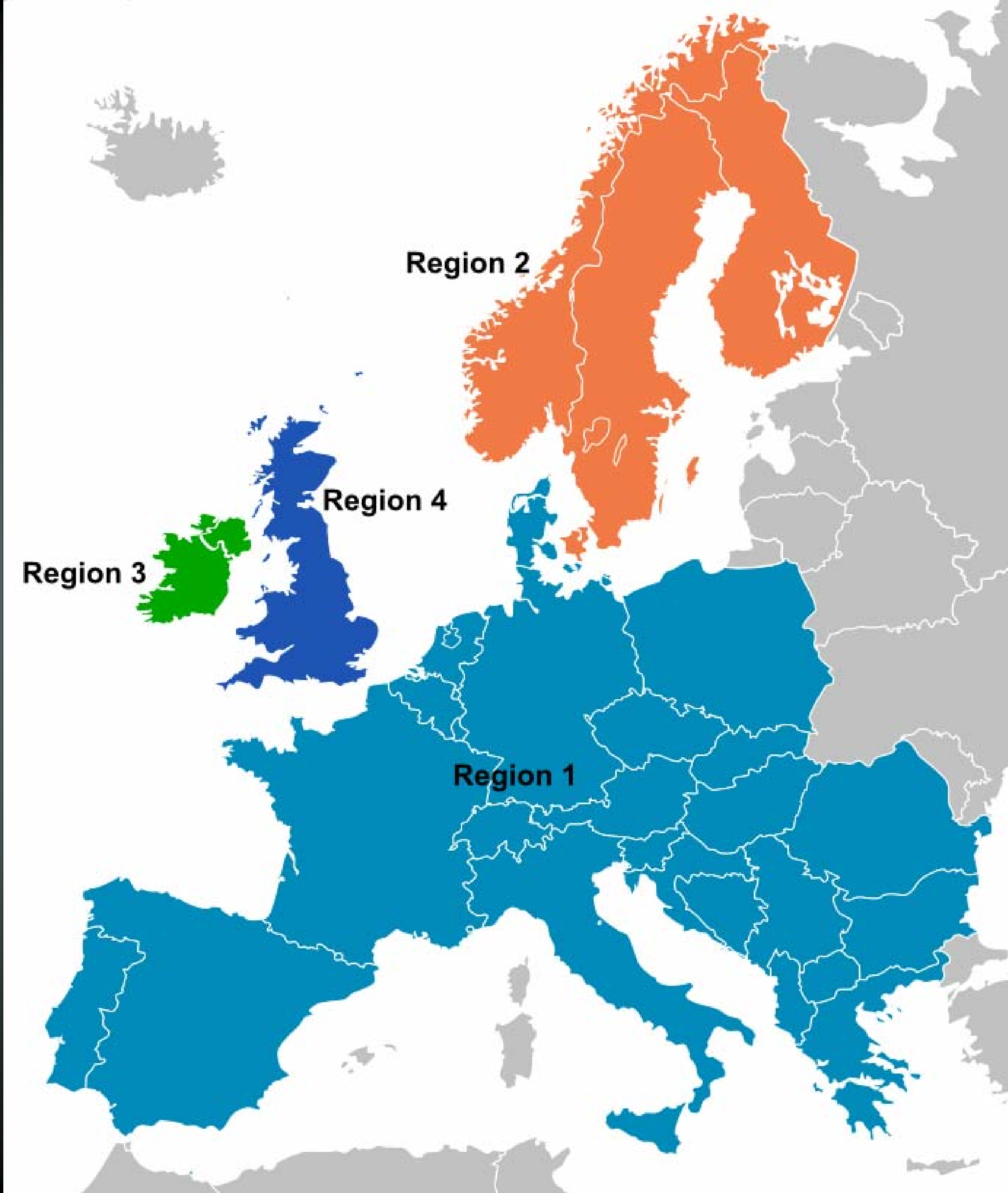
(a) Circular structure (b) Symmetric structure (c) General structure

Fig. 1. Three types of topological structures. The darker is the node, the higher the topological centrality is. The black nodes are the topological centers. Networks of circular structure have n $(n \geq 3)$ topological centers; network of symmetric structure has 2 topological centers; other networks have 1 topological center.

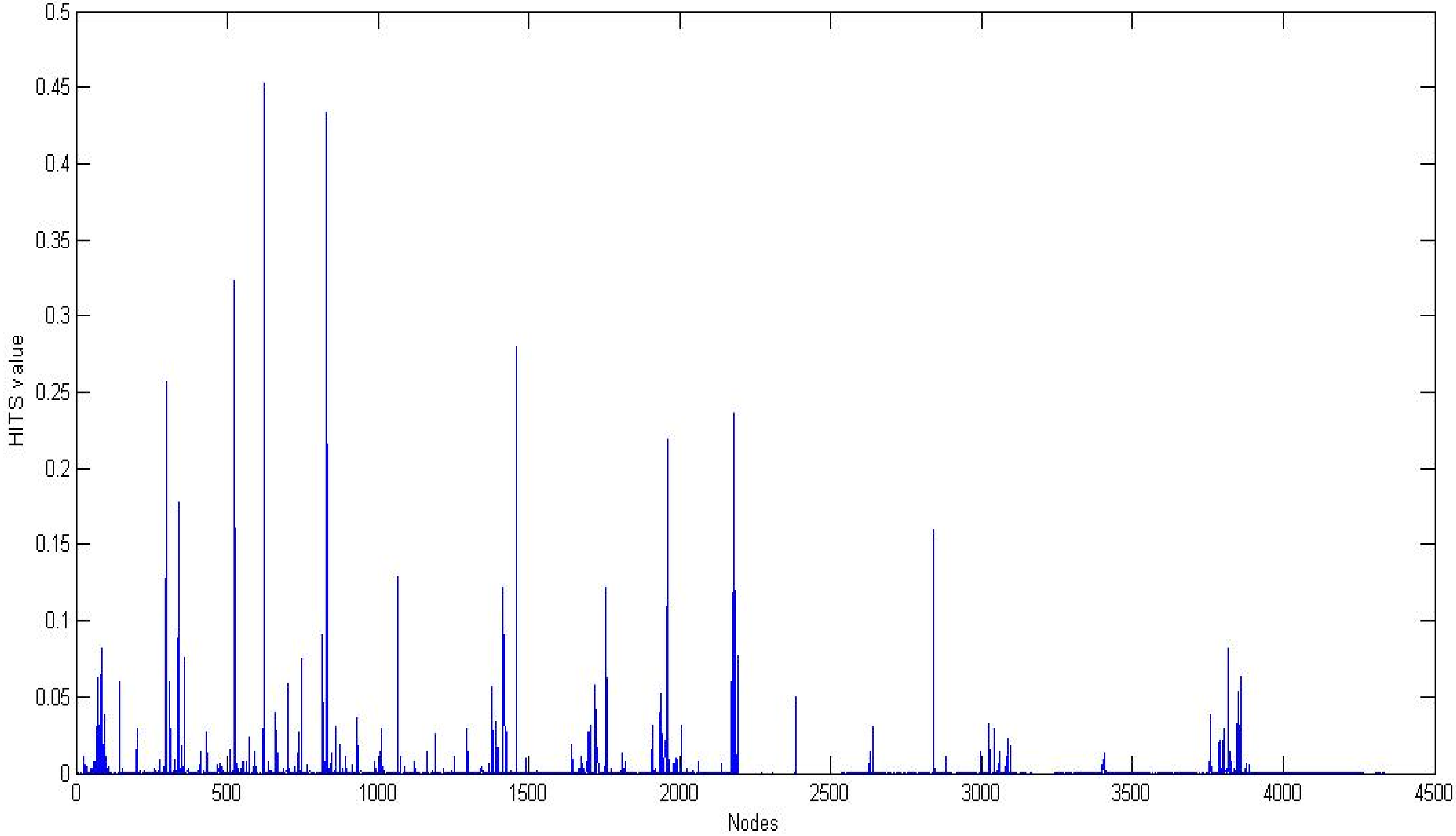
The EU power grid

- UCTE associates most of the continental Europe national power grid operators in order to coordinate the production and demand of some annual 2,300 TWh and 450 million customers from 24 countries.
- The map gives data from the transmission network (voltage levels from 110 kV to 400 kV) and ignores the much more extended distribution one. Nonetheless, it deals with more than 3,000 nodes (generators and substations) and some 200,000 km of transmission lines.

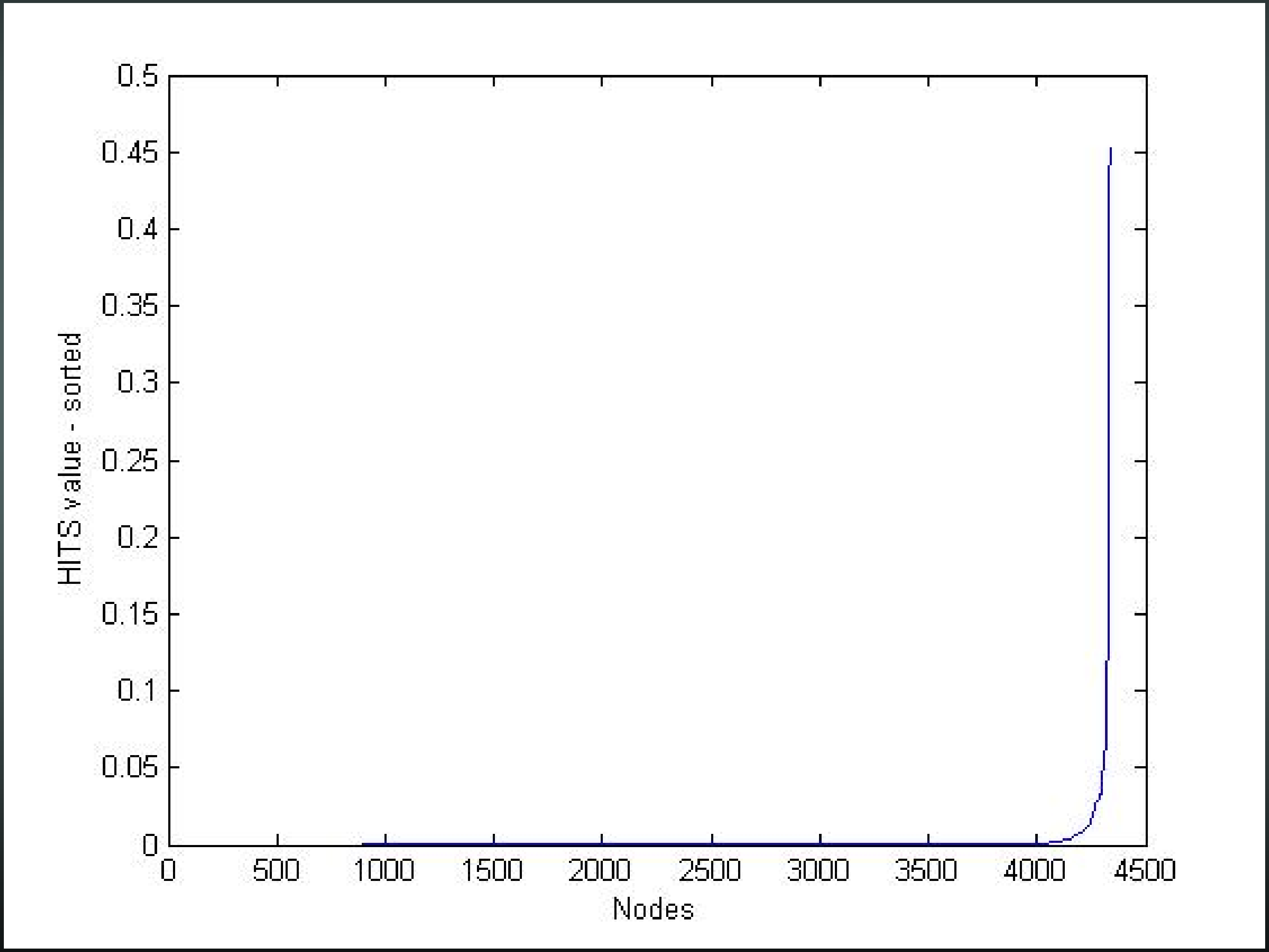
- The experimental dataset contains the electricity lines above 200kV grouped by disconnected regions ([1] Main Europe, [2] Nordic Countries, [3] Ireland, [4] UK)
- Only region 1 was observed



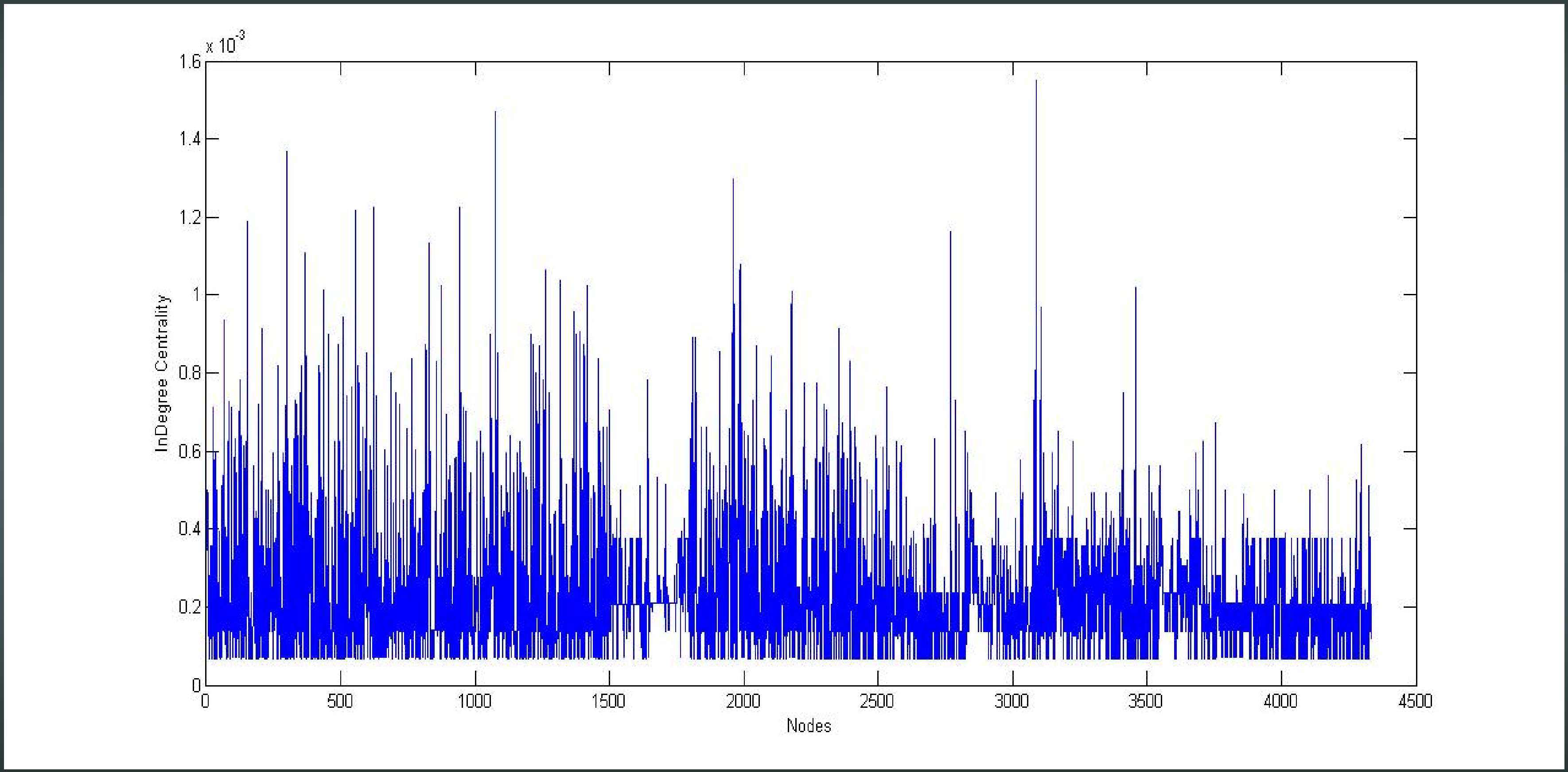
HITS - Authorities



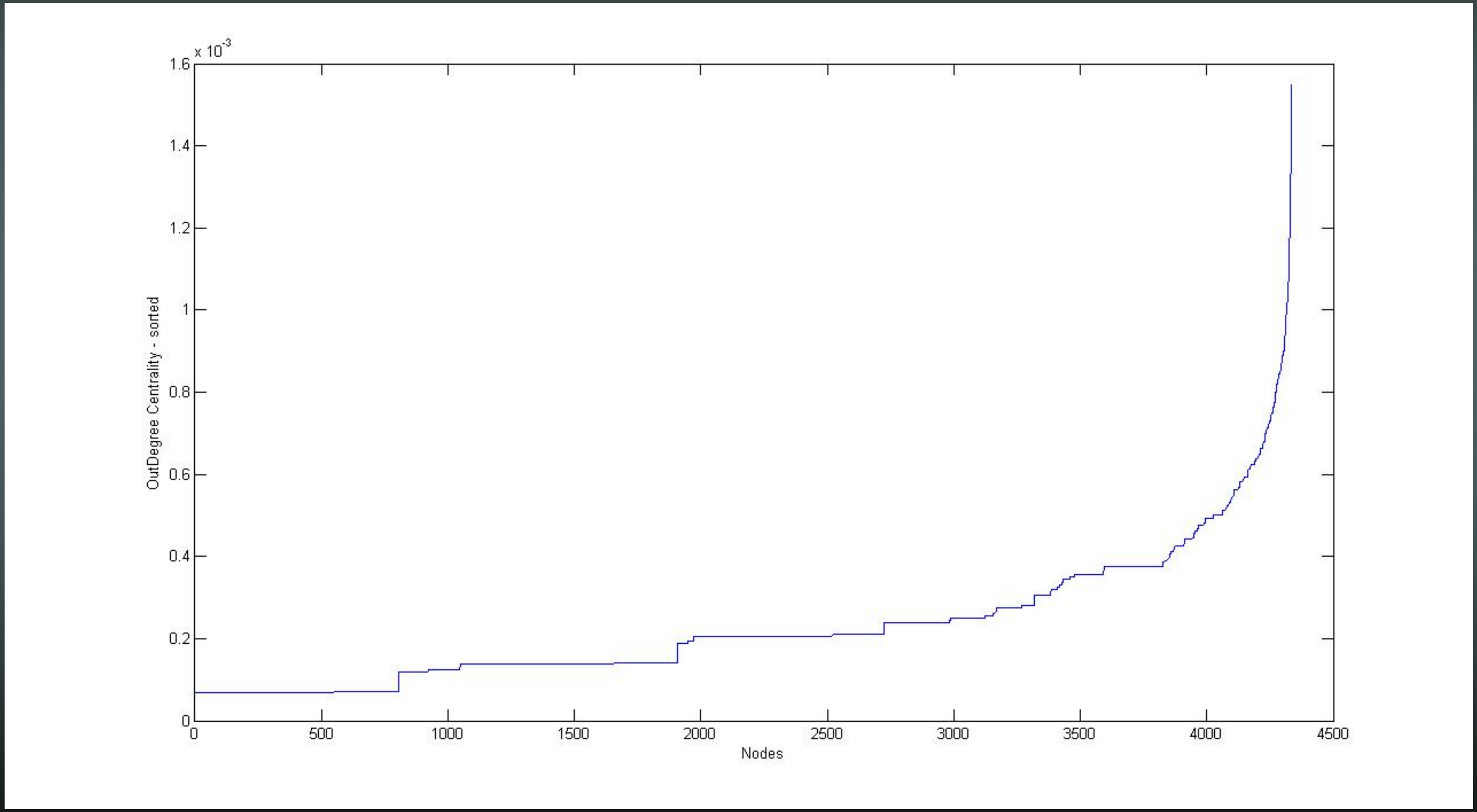
HITS – Authorities (sorted)



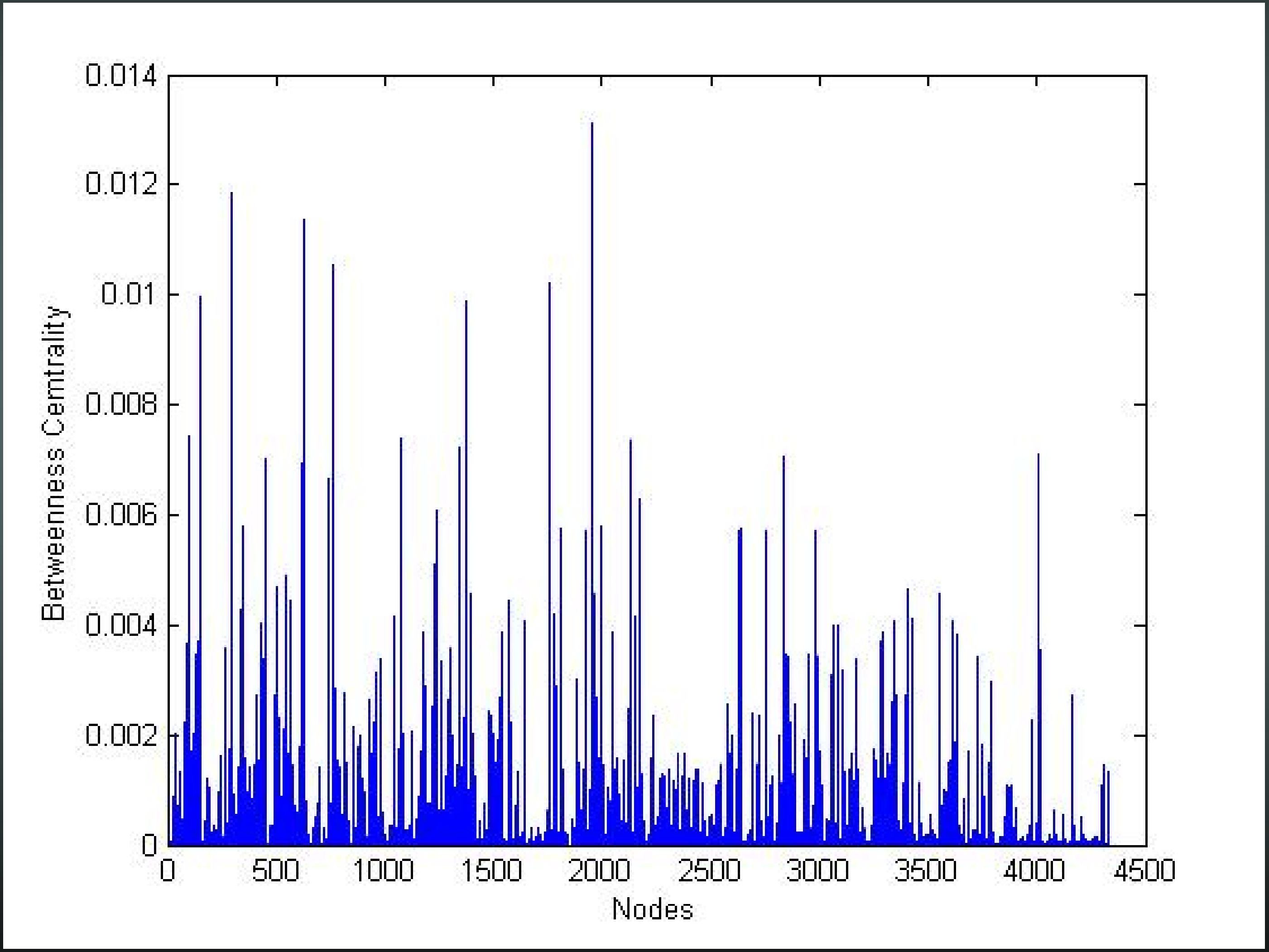
Degree Centrality



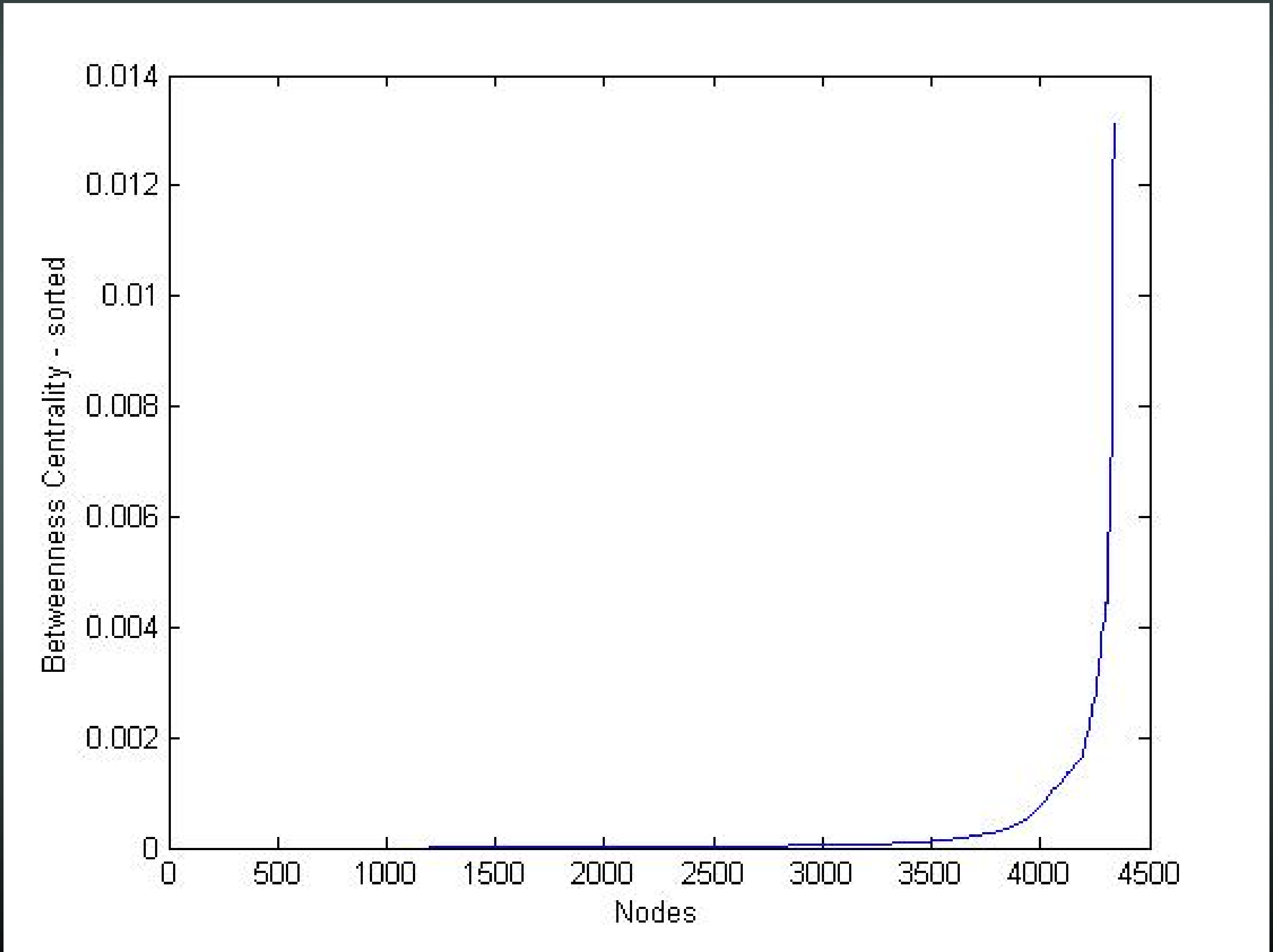
Degree Centrality (Sorted)



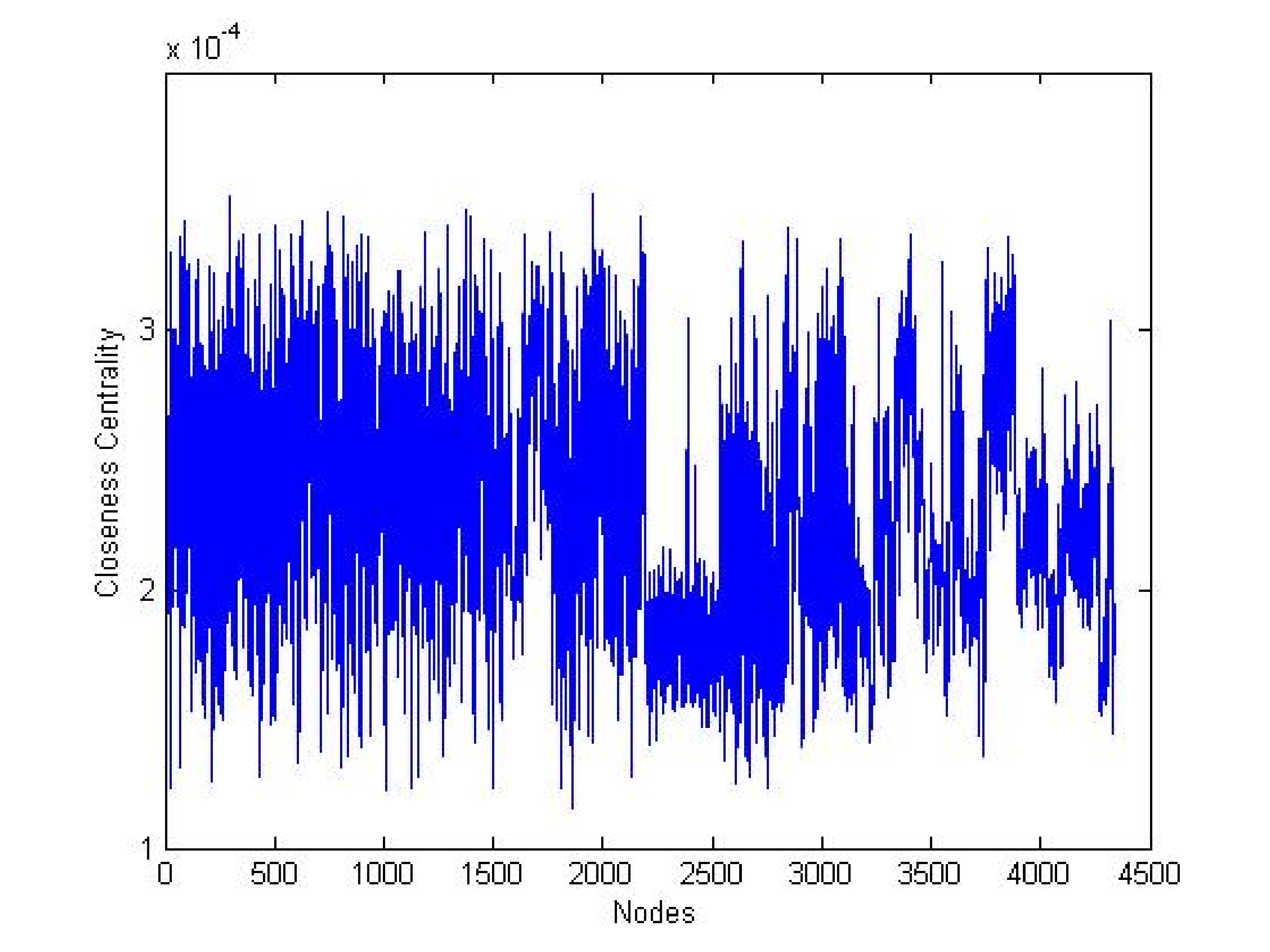
Betweenness Centrality



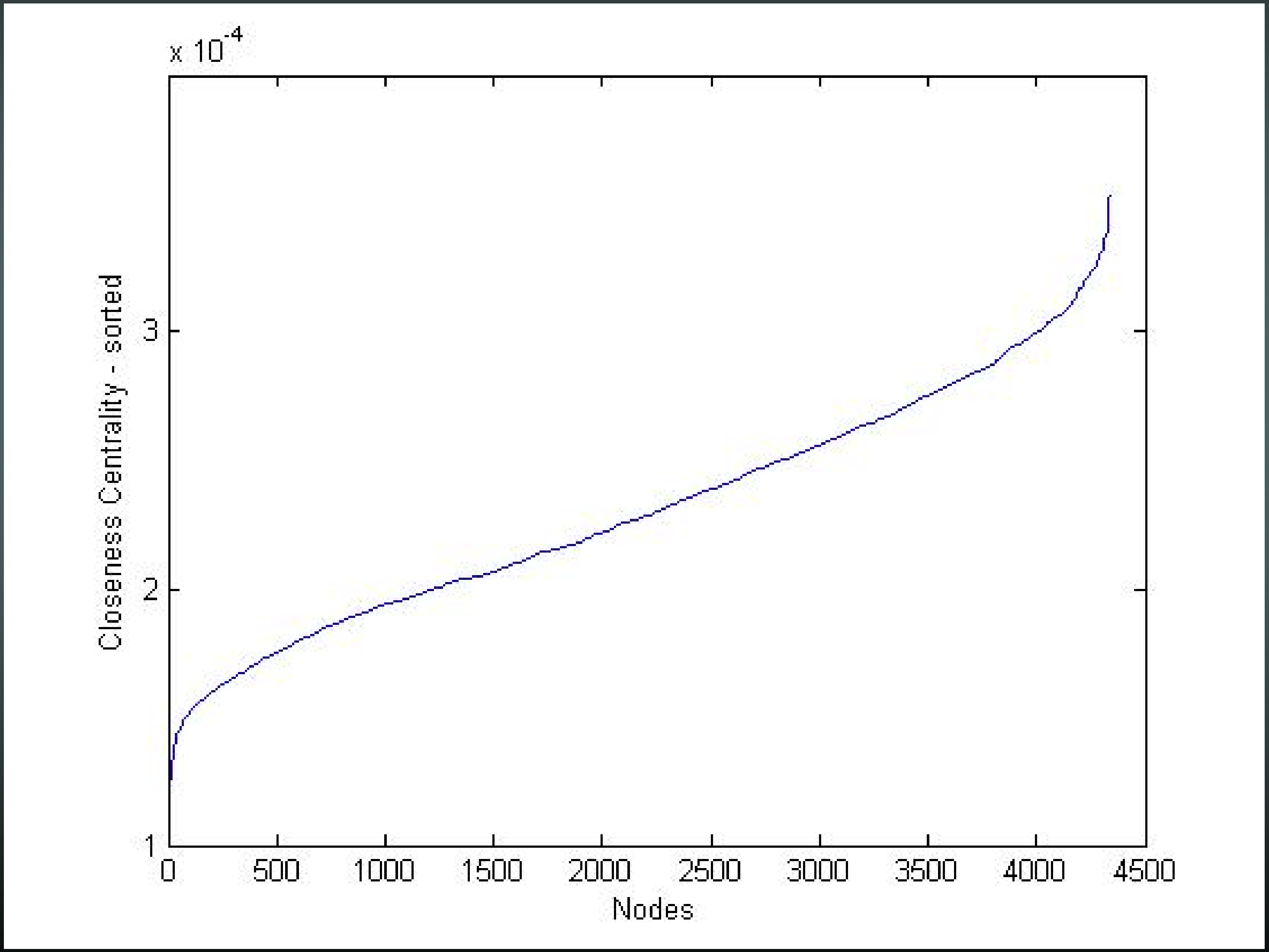
Betweenness Centrality (sorted)



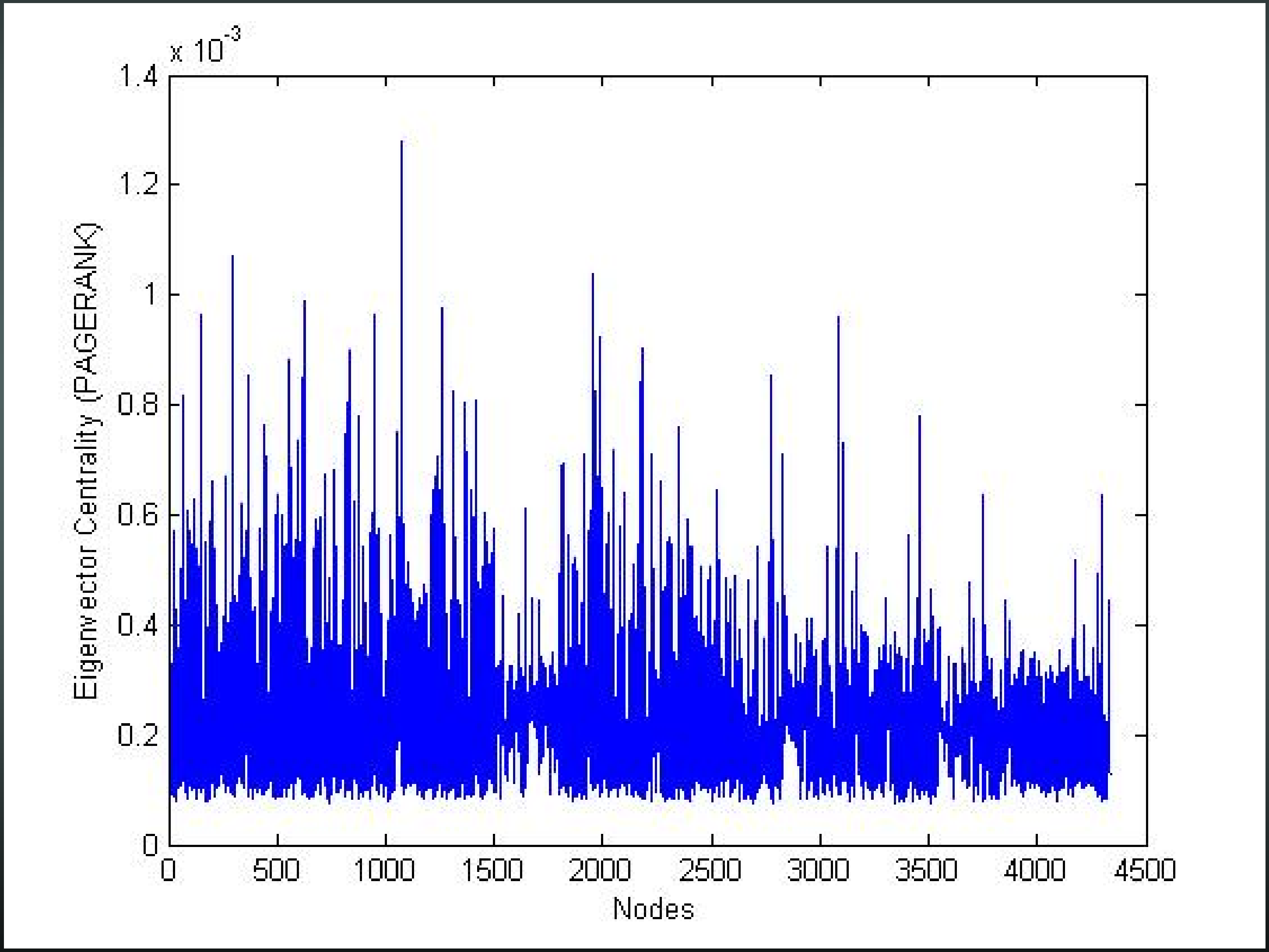
Closeness Centrality



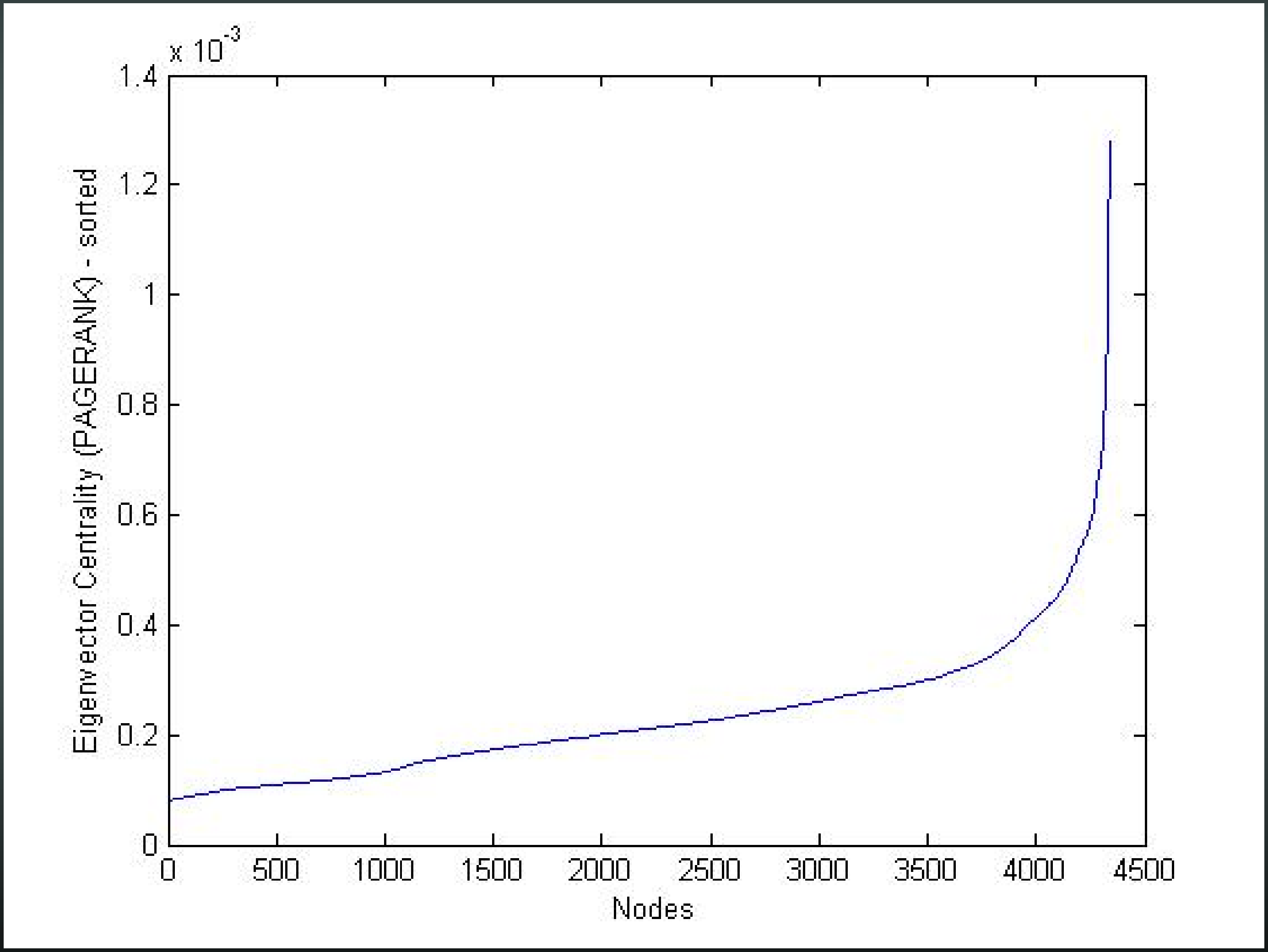
Closeness Centrality (sorted)



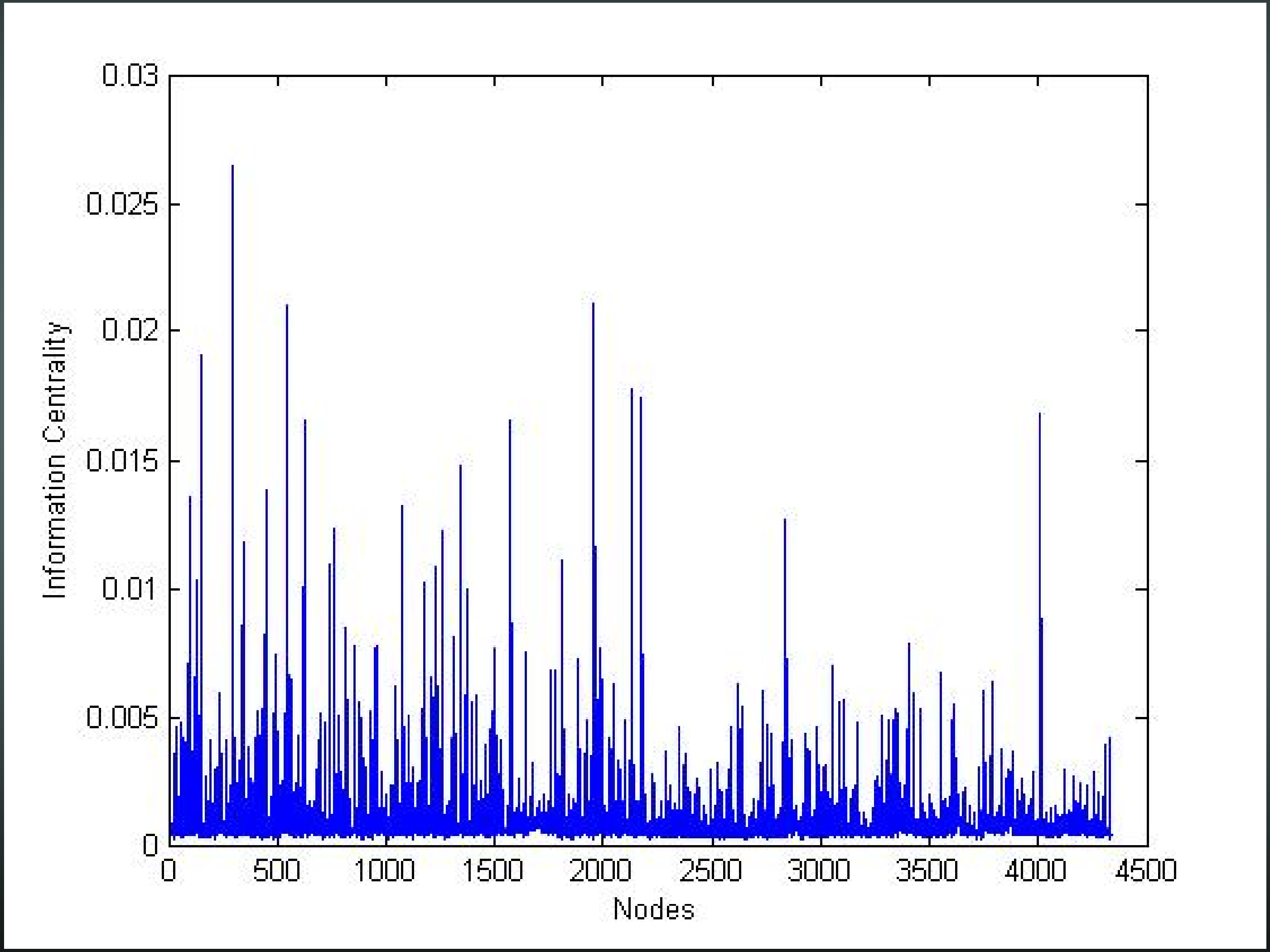
Eigenvector centrality



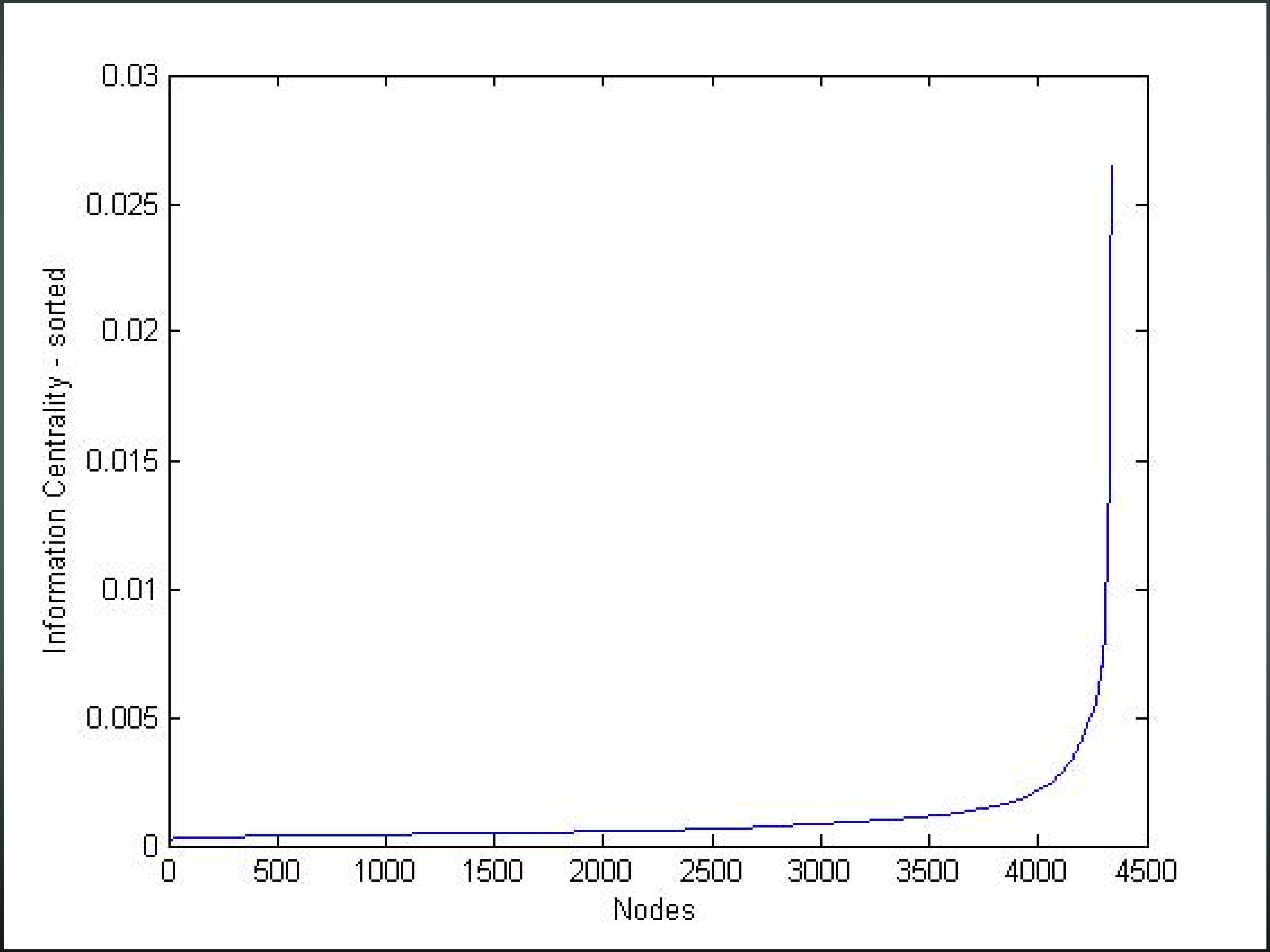
Eigenvector centrality (sorted)



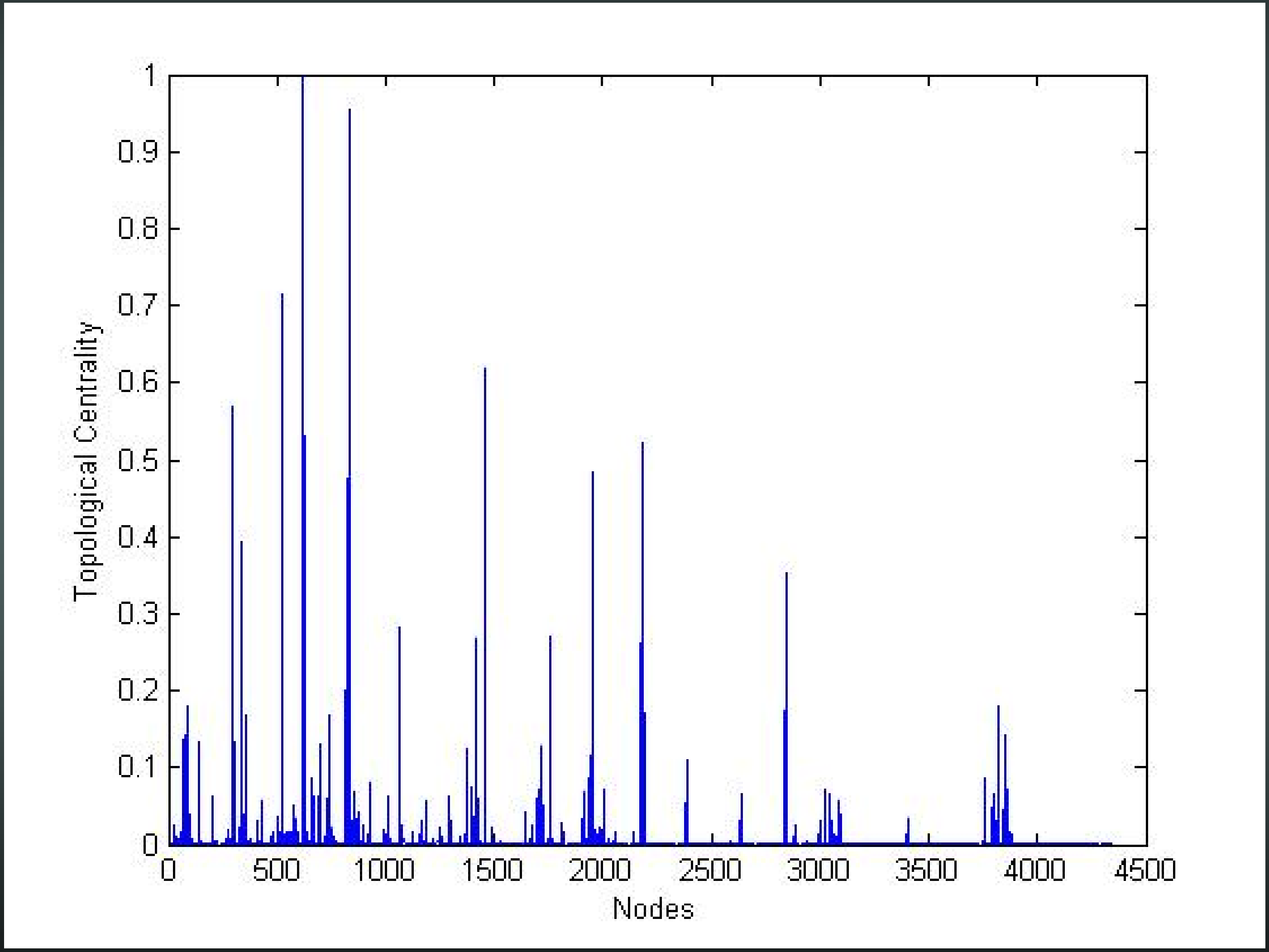
Information Centrality



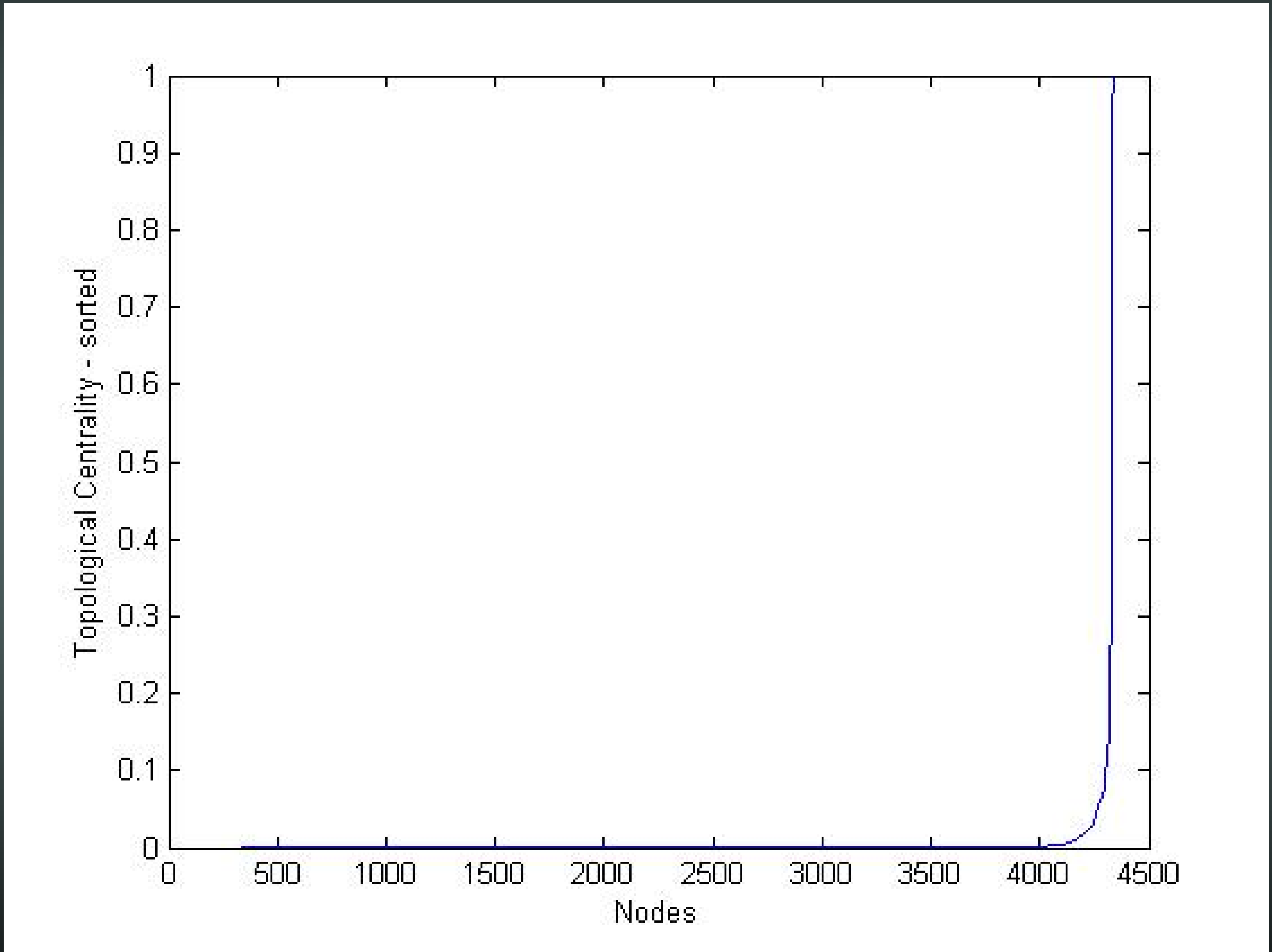
Information Centrality (sorted)



Topological Centrality



Topological Centrality (sorted)



FINDING THE FIRST 10 NODES BY EACH CRITERION

	HITS Authority		Degree Centrality		Betweenness Centrality		Closeness Centrality		PageRank		Information Centrality		Topological Centrality	
	Node	Val	Node	Val	Node	Val	Node	Val	Node	Val	Node	Val	Node	Val
1	622	0,4526	3086	0,0015	1958	0,0131	1958	0,00035	1074	0,0013	297	0,0264	622	1
2	830	0,4327	1074	0,0015	297	0,0119	297	0,00035	297	0,0011	1958	0,0211	830	0,956
3	524	0,3231	297	0,0014	623	0,0114	1378	0,00035	1958	0,001	543	0,021	524	0,7139
4	1459	0,2798	1958	0,0013	765	0,0105	745	0,00035	623	0,00098	151	0,0191	1459	0,6183
5	297	0,2573	623	0,0012	1756	0,0102	1391	0,00034	1261	0,00097	2131	0,0178	297	0,5688
6	2181	0,2363	944	0,0012	151	0,01	2173	0,00034	151	0,00096	2173	0,0174	2181	0,5221
7	1958	0,2195	554	0,0012	1378	0,0099	816	0,00034	944	0,00096	4012	0,0168	1958	0,4853
8	338	0,178	151	0,0012	94	0,0074	83	0,00034	3086	0,00095	623	0,0166	338	0,3934
9	2842	0,16	2769	0,0012	1072	0,0074	623	0,00034	1989	0,00092	1576	0,0165	2842	0,3536
10	1065	0,1283	830	0,0011	2131	0,0073	508	0,00034	2181	0,00090	1339	0,0148	1065	0,2834

References (1)

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References (2)

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